Answers
Teacher Copy

Lesson 10-2 Writing a Quadratic Function Given Three Points

Plan

Pacing: 1 class period

Chunking the Lesson

Example A #1 Example B

Example C #2

Check Your Understanding

Lesson Practice

Teach

Bell-Ringer Activity

Students should recall that an absolute value of a number is its distance from zero on a number line.

Have students evaluate the following:

1. \(|6| \ [6]
2. \(|–6| \ [6]

Then have students solve the following equation.

3. \(|x|= 6 \ [x = 6 \text{ or } x = –6]

Example A Marking the Text, Interactive Word Wall

Point out the Math Tip to reinforce why two solutions exist. Work through the solutions to the equation algebraically. Remind students that...
solutions to an equation make the equation a true statement. This mathematical understanding is necessary for students to be able to check their results.

**Developing Math Language**

An *absolute value equation* is an equation involving an absolute value of an expression containing a variable. Just like when solving algebraic equations without absolute value bars, the goal is to isolate the variable. In this case, isolate the absolute value bars because they contain the variable. It should be emphasized that when solving absolute value equations, students must think of two cases, as there are two numbers that have a specific distance from zero on a number line.

**1 Identify a Subtask, Quickwrite**

When solving absolute value equations, students may not see the purpose in creating two equations. Reviewing the definition of the absolute value function as a piecewise-defined function with two rules may enable students to see the reason why two equations are necessary.

Have students look back at Try These A, parts c and d. Have volunteers construct a graph of the two piecewise-defined functions used to write each equation and then discuss how the solution set is represented by the graph.

**Example B Marking the Text, Simplify the Problem, Critique Reasoning, Group Presentation**

Start with emphasizing the word *vary* in the Example, discussing what it means when something varies. You may wish to present a simpler example such as: The average cost of a pound of coffee is $8. However, the cost sometimes varies by $1. This means that the coffee could cost as little as $7 per pound or as much as $9 per pound. Now have students work in small groups to examine and solve Example B by implementing an absolute value equation. Additionally, ask them to take the problem a step further and graph its solution on a number line. Have groups present their findings to the class.

**ELL Support**

For those students for whom English is a second language, explain that the word *varies* in mathematics means *changes*. There are different ways of thinking about how values can vary. Values can vary upward or downward, less than or greater than, in a positive direction or a negative direction, and so on. However, the importance comes in realizing that there are two different directions, regardless of how you think of it.

Also address the word *extremes* as it pertains to mathematics. An extreme value is a maximum value if it is the largest possible amount (greatest value), and an extreme value is a minimum value if it is the smallest possible amount (least value).

**Developing Math Language**

An *absolute value inequality* is basically the same as an absolute value equation, except that the equal sign is now an inequality symbol: <,
>, ≤, ≥, or ≠. It still involves an absolute value expression that contains a variable, just like before. Use graphs on a number line of the solutions of simple equations and inequalities and absolute value equations and inequalities to show how these are all related.

**Example C Simplify the Problem, Debriefing**

Before addressing Example C, discuss the following: Inequalities with \(|A| > b\), where \(b\) is a positive number, are known as *disjunctions* and are written as \(A < -b\) or \(A > b\).

For example, \(|x| > 5\) means the value of the variable \(x\) is more than 5 units away from the origin (zero) on a number line. The solution is \(x < -5\) or \(x > 5\).

See graph A.

This also holds true for \(|A| \geq b\).

Inequalities with \(|A| < b\), where \(b\) is a positive number, are known as *conjunctions* and are written as \(-b < A < b\), or as \(-b < A\) and \(A < b\).

For example: \(|x| < 5\); this means the value of the variable \(x\) is less than 5 units away from the origin (zero) on a number line. The solution is \(-5 < x < 5\).

See graph B.

This also holds true for \(|A| \leq b\).

Students can apply these generalizations to Example C. Point out that they should proceed to solve these just as they would an algebraic equation, except in two parts, as shown above. After they have some time to work through parts a and b, discuss the solutions with the whole class.

**Teacher to Teacher**

Another method for solving inequalities relies on the geometric definition of absolute value \(|x - a|\) as the distance from \(x\) to \(a\). Here’s how you can solve the inequality in the example:

Thus, the solution set is all values of \(x\) whose distance from is greater than . The solution can be represented on a number line and written as \(x < -4\) or \(x > 1\).

**2 Quickwrite, Self Revision/Peer Revision, Debriefing**

Use the investigation regarding the restriction \(c > 0\) as an opportunity to discuss the need to identify impossible situations involving
inequalities.

**Check Your Understanding**

Debrief students’ answers to these items to ensure that they understand concepts related to absolute value equations. Have groups of students present their solutions to Item 4.

**Assess**

Students’ answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

**Adapt**

Check students’ answers to the Lesson Practice to ensure that they understand basic concepts related to writing and solving absolute value equations and inequalities and graphing the solutions of absolute value equations and inequalities. If students are still having difficulty, review the process of rewriting an absolute value equation or inequality as two equations or inequalities.

**Plan**

**Pacing:** 1 class period

**Chunking the Lesson**

#1–3   #4–6   #7–10

Check Your Understanding

Lesson Practice

**Teach**

**Bell-Ringer Activity**

Have students write the equation in slope-intercept form of a line that passes through the points (3, 2) and (−5, 6), using the following procedures:
1. Substitute the point (3, 2) into the equation $y = mx + b$.
2. Substitute the point $(-5, 6)$ into the equation $y = mx + b$.
3. Solve the system of equations using substitution or Gaussian elimination.

1–3 Interactive Word Wall, Create Representations

Encourage the use of proper math vocabulary to describe the similarities and differences of the three parabolas. Students could use the terms vertex, axis of symmetry, maximum, minimum, y-intercept, and x-intercepts in their descriptions.

Universal Access

Students may question where the equations $y = 2x^2 - 14x + 10$ and $y = -x^2 + 7x - 10$ in Item 2b came from. Return to Item 1 and lead students through writing binomial factors other than $(x - 2)$ and $(x - 5)$ that lead to a quadratic equation with solution set $\{2, 5\}$.

4–6 Predict and Confirm, Create Representations, Visualization

Ask students to plot the three points on a coordinate plane and make predictions about the vertex of the parabola. Have students share answers and note that it is impossible to accurately predict the vertex. This will motivate the algebraic solution process.

Universal Access

Using algebraic methods to solve a system of equations in three variables can be time-consuming and frustrating to students. Consider allowing students to use matrix equations and their graphing calculators to determine the values of $a$, $b$, and $c$. Doing this will keep the lesson focused on finding the equation of the parabola.

7–10 Debriefing, Identify a Subtask, Summarizing

Ask students to provide a summary of the method used to determine the equation of a parabola in standard form that passes through three given points.

Teacher to Teacher

If an additional example is needed, students can create examples for a partner by working backward. Have students graph a parabola in standard form on a calculator and use the table or trace function to identify three integral points that lie on the parabola. Students can exchange points and then check each other’s work.

Check Your Understanding
Debrief students’ answers to these items to ensure that they understand concepts related to writing equations of quadratic functions.

**Assess**

Students’ answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

**Adapt**

Check students’ answers to the Lesson Practice to ensure that they understand how to write the equation of a parabola in standard form given three points that lie on the parabola. For additional practice, students can make up their own problems. Have them select three points and write the equation of the parabola using the method learned in this activity. Show students how they can check their work using quadratic regression on their graphing calculators.

**Learning Targets**

- Explain why three points are needed to determine a parabola.
- Determine the quadratic function that passes through three given points on a plane.

**Create Representations (Learning Strategy)**

**Definition**

Creating pictures, tables, graphs, lists, equations, models, and/or verbal expressions to interpret text or data

**Purpose**

Helps organize information using multiple ways to present data and to answer a question or show a problem solution

**Quickwrite (Learning Strategy)**

**Definition**

Writing for a short, specific amount of time about a designated topic
Purpose

Helps generate ideas in a short time

Create Representations (Learning Strategy)

Definition

Creating pictures, tables, graphs, lists, equations, models, and/or verbal expressions to interpret text or data

Purpose

Helps organize information using multiple ways to present data and to answer a question or show a problem solution

Identify a Subtask (Learning Strategy)

Definition

Breaking a problem into smaller pieces whose outcomes lead to a solution

Purpose

Helps to organize the pieces of a complex problem and reach a complete solution

Suggested Learning Strategies

Create Representations, Quickwrite, Questioning the Text, Create Representations, Identify a Subtask

Recall that if you are given any two points on the coordinate plane, you can write the equation of the line that passes through those points. The two points are said to determine the line because there is only one line that can be drawn through them.

Do two points on the coordinate plane determine a parabola? To answer this question, work through the following items.
Math Tip

To review writing a quadratic equation when given its solutions, see Lesson 7-3.

1. Follow these steps to write the equation of a quadratic function whose graph passes through the points (2, 0) and (5, 0).

   a. Write a quadratic equation in standard form with the solutions $x = 2$ and $x = 5$.

      $x^2 - 7x + 10 = 0$ or a nonzero multiple of this equation

   b. Replace 0 in your equation from part a with $y$ to write the corresponding quadratic function.

      Answers may vary depending on the equation in part a. Sample answer: $y = x^2 - 7x + 10$

   c. Use substitution to check that the points (2, 0) and (5, 0) lie on the function's graph.

      \[
      \begin{align*}
      0^2(2) - 7(2) + 10 &= 0^2(5) - 7(5) + 10 \\
      0 - 14 + 10 &= 0 - 35 + 10 \\
      0 &= 0 \checkmark \\
      0 &= 0 \checkmark
      \end{align*}
      \]

2.

   a. Use appropriate tools strategically. Graph your quadratic function from Item 1 on a graphing calculator.

      Check students' work.

   b. On the same screen, graph the quadratic functions $y = 2x^2 - 14x + 20$ and $y = -x^2 + 7x - 10$.

      Check students' work.
c. Describe the graphs. Do all three parabolas pass through the points (2, 0) and (5, 0)?

Answers may vary, but students should note that all three parabolas pass through the points (2, 0) and (5, 0).

Sample answer: Two of the parabolas open upward, and one opens downward. One parabola is narrower than the others. However, all of the parabolas have the same x-intercepts: 2 and 5.


No. Sample explanation: My graph of the three parabolas shows that more than one parabola can be drawn through the same pair of points, (2, 0) and (5, 0). So, two points are not enough to determine a parabola.

Math Tip

Three or more points are collinear if they lie on the same straight line.

Three points in the coordinate plane that are not on the same line determine a parabola given by a quadratic function. If you are given three noncollinear points on the coordinate plane, you can write the equation of the quadratic function whose graph passes through them.

Consider the quadratic function whose graph passes through the points (1, 2), (3, 0), and (5, 6).

4. Write an equation by substituting the coordinates of the point (1, 2) into the standard form of a quadratic function, \(y = ax^2 + bx + c\).

\[2 = a + b + c\] or equivalent

5. Write a second equation by substituting the coordinates of the point (3, 0) into the standard form of a
quadratic function.

\[ 0 = 9a + 3b + c \] or equivalent

6. Write a third equation by substituting the coordinates of the point \((5, 6)\) into the standard form of a quadratic function.

\[ 6 = 25a + 5b + c \] or equivalent

7. Use your equations from Items 4–6 to write a system of three equations in the three variables \(a, b,\) and \(c\).

\[
\begin{align*}
  a + b + c &= 2 \\
  9a + 3b + c &= 0 & \text{or equivalent} \\
  25a + 5b + c &= 6
\end{align*}
\]

Math Tip

To review solving a system of three equations in three variables, see Lesson 3-2.

8. Use substitution or Gaussian elimination to solve your system of equations for \(a, b,\) and \(c\).

\[ a = 1, \ b = -5, \ c = 6 \]

9. Now substitute the values of \(a, b,\) and \(c\) into the standard form of a quadratic function.

\[ y = x^2 - 5x + 6 \]
10. **Model with mathematics.** Graph the quadratic function to confirm that it passes through the points (1, 2), (3, 0), and (5, 6).

**Check Your Understanding**

11. **Describe how to write the equation of a quadratic function whose graph passes through three given points.**

Substitute the coordinates of each point into the standard form of a quadratic function, \( y = ax^2 + bx + c \). Write the 3 resulting equations as a system of equations. Then solve the system for the values of \( a, b, \) and \( c \). Finally, use the values of \( a, b, \) and \( c \) to write the equation of the quadratic function in standard form.

12. 
   a. **What happens when you try to write the equation of the quadratic function that passes through the points (0, 4), (2, 2), and (4, 0)?**

   You find that \( a = 0, b = -1, \) and \( c = 4 \), which results in the function \( f(x) = -x + 4 \). This function is linear, not quadratic.

   b. **What does this result indicate about the three points?**

   The 3 points are on the same line, which means that you cannot write the equation of a quadratic function whose graph passes through the points.

13. 
   a. **Reason quantitatively.** The graph of a quadratic function passes through the point (2, 0). The vertex of the graph is \((-2, -16)\). Use symmetry to identify another point on the function's graph. Explain how you determined your answer.

   \((-6, 0)\). Sample explanation: For a quadratic function, the axis of symmetry is a vertical line that passes through the vertex, so the axis of symmetry is \((x = -2)\). The point \((2, 0)\) is 4 units to the right of the axis of symmetry, so there will be another point on the graph of the function 4 units to the left of the axis of symmetry with the same y-coordinate. This point has coordinates \((-6, 0)\).
### Lesson 10-2 Practice

Write the equation of the quadratic function whose graph passes through each set of points.

14. \((-3, 2), (-1, 0), (1, 6)\)

\[ y = x^2 + 3x + 2 \]

15. \((-2, -5), (0, -3), (1, 4)\)

\[ y = 2x^2 + 5x - 3 \]

16. \((-1, -5), (1, -9), (4, 0)\)

\[ y = x^2 - 2x - 8 \]

17. \((-3, 7), (0, 4), (1, 15)\)

\[ y = 3x^2 + 8x + 4 \]

18. \((1, 0), (2, -7), (5, -16)\)

\[ y = x^2 + 4x - 12 \]
\( y = x^2 - 10x + 9 \)

19. \((-2, -11), (-1, -12), (1, 16)\)

\( y = 5x^2 + 14x - 3 \)

**Math Tip**

A sequence is an ordered list of numbers or other items. Each number or item in a sequence is called a term.

20. The table below shows the first few terms of a sequence. This sequence can be described by a quadratic function, where \( f(n) \) represents the \( n \)th term of the sequence. Write the quadratic function that describes the sequence.

<table>
<thead>
<tr>
<th>Term Number, ( n )</th>
<th>Term of Sequence, ( f(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
</tr>
</tbody>
</table>

\( f(n) = n^2 + n \)
Connect to Geometry

A regular hexagon is a six-sided polygon with all sides having the same length and all angles having the same measure.

21. A quadratic function $A(s)$ gives the area in square units of a regular hexagon with a side length of $s$ units.

a. Use the data in the table below to write the equation of the quadratic function.

<table>
<thead>
<tr>
<th>Side Length, $s$</th>
<th>Area, $A(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$6\sqrt{3}$</td>
</tr>
<tr>
<td>4</td>
<td>$24\sqrt{3}$</td>
</tr>
<tr>
<td>6</td>
<td>$54\sqrt{3}$</td>
</tr>
</tbody>
</table>

$A(s) = \frac{3\sqrt{3}}{2} s^2$

b. Attend to precision. To the nearest square centimeter, what is the area of a regular hexagon with a side length of 8 cm?

$166\, \text{cm}^2$