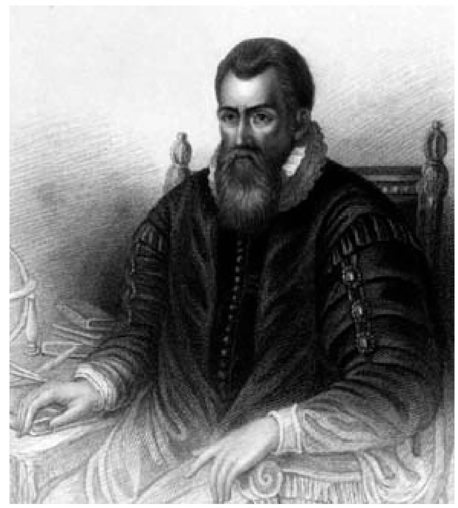
Algebra II/Trig Name ____________________

Properties of Logarithms
Module 2, Unit 4, Lesson 5

History of Logarithms

Long before the calculator, logarithms were great mathematical labor-saving devices!

Although there is evidence that logarithms were known in 8th century India, their invention as an aid to calculation is attributed to a Scottish nobleman named John Napier (1550-1617) . . . In collaboration with Oxford professor Henry Briggs, Napier refined his logarithms by constructing tables for logarithms in base 10. . .



Napier lived during an age of great innovation in the world of astronomy. Copernicus had published his theory of the solar system in 1543, and many astronomers were eagerly involved in calculating and re-calculating planetary positions based in the wake of Copernicus's ideas. Their calculations took up pages and pages and hours and hours of work. Johannes Kepler (1571-1630) still had to fill nearly 1000 large pages with dense arithmetical computations to obtain his famous laws of planetary motions! Napier's logarithms helped ease that burden.

Because they are exponents, logarithms allow tedious calculations (like multiplying and dividing very large numbers) to be replaced by the simpler process of adding and subtracting the corresponding logarithms.

Not that mathematicians simply put down their pens after Napier. Many objected to using logarithms because no one knew understood they worked (an objection similar to one made to the use of computers in the 1960s)!

[http://ualr.edu/lasmoller/napier.html]

Logarithmic Functions

A ***logarithm function*** is a function in form of $f(x) = \log_b x$**,** where $x > 0$, b > 0 and $b \neq 1$.

It is always easy to change from an exponential function to logarithm function because

$$y = \log_b x \text{ is equivalent to } x = b^y$$

Domain of a logarithmic function is x > 0.

Example 1: Write each equation in its equivalent exponential form:

a. $2 = \log_5 x$ b. $3 = \log_b 64$ c. $\log_3 7 = y$

Example 2: Write each equation in its equivalent logarithmic form:

a. $12^2 = x$ b. $b^3 = 8$ c. $e^y = 9$

Common Logarithmic Functions

The logarithmic function with base 10 is called the ***common logarithmic function.***

$$y = \log_{10} x \quad \text{is written} \quad y = \log x$$

Natural Logarithmic Functions

The logarithmic function with base *e* is called the ***natural logarithmic function.***

$$y = \log_e x \quad \text{is written} \quad y = \ln x$$

The Natural Base *e*

The number *e* is defined as the value that $\left(1+\frac{1}{n}\right)^n$ gets closer to as n gets larger and larger. As $n \to \infty$**,** then $\left(1+\frac{1}{n}\right)^n \to e$**.** The value of $e \approx 2.718281827.....$

Properties of Logarithms

Base b	Base 10	Base e
$\log_b 1 = 0$	$\log\ 1 = 0$	$\ln\ 1 = 0$
$\log_b b = 1$	$\log\ 10 = 1$	$\ln\ e = 1$
$\log_b b^x = x$	$\log\ 10^x = x$	$\ln e^x = x$
$b^{\log_b x} = x$	$10^{\log x} = x$	$e^{\ln x} = x$

Example 3: Evaluate each expression without using a calculator.

a. $\log_2 16$ b. $7^{\log_7 23}$ c. $\log_3 \frac{1}{\sqrt{3}}$

d. $\log_4 4^5$

e. $10^{\log \sqrt{x}}$

f. $\log 10^7$

g. $\ln \frac{1}{e^7}$

h. $\ln e^{13x}$

i. $\log 1000$

j. $e^{\ln 125}$

k. $\log_5 625$

l. $\log_2 \left(\frac{1}{32}\right)$

Properties of Logarithms

Product Rule	$\log_b(MN) = \log_b M + \log_b N$
Quotient Rule	$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$
Power Rule	$\log_b M^p = p \log_b M$

Example 4: Use properties of logarithms to expand each logarithmic expression as much as possible.

a. $\log_5(7 \bullet 3)$

b. $\log_7\left(\frac{7}{x}\right)$

c. $\log_b x^3$

d. $\log_5\left(\frac{\sqrt{x}}{25}\right)$

e. $\log_2 \sqrt[5]{\dfrac{xy^4}{16}}$

f. $\log\left[\dfrac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right]$

g. $\log_8\left(\dfrac{64}{\sqrt{x+1}}\right)$

h. $\ln\left[\dfrac{x^3\sqrt{x^2+1}}{(x+1)^4}\right]$

Example 5: Use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1.

a. $5\ln x - 2\ln y$

b. $\dfrac{1}{2}(\log x + \log y)$

c. $\log(2x+5) - \log x$

d. $\dfrac{1}{3}(\log_4 x - \log_4 y) + 2\log_4(x+1)$

e. $\log x + \log(x^2-1) - \log 7 - \log(x+1)$

f. $4\ln x + 7\ln y - 3\ln z$

g. $\frac{1}{3}(\log_4 x - \log_4 y)$

h. $2\ln x - \frac{1}{2}\ln y$

Change-of-Base Property $\log_b M = \frac{\log_a M}{\log_a b}$

Example 6: Use common logarithms or natural logarithms to evaluate to four decimal places.

a. $\log_5 13$

b. $\log_{14} 87.5$