Algebra II/Trig Name ______________________

Exponential Growth and Decay Notes
Module 2, Unit 4, Lesson 8

Background

Exponential functions model data that have a constant percent increase or decrease. This should not be confused with a constant rate of change, which linear functions have. Informally, a constant percent increase or decrease guarantees that the data will grow or decay rapidly.

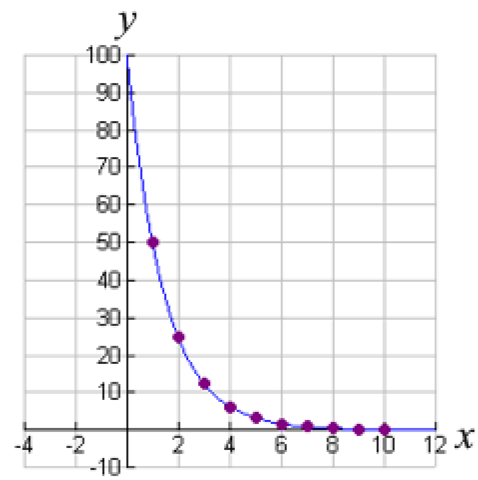
Many real world phenomena can be modeled by functions that describe how things grow or decay as time passes. Examples of such phenomena can include the studies of populations, bacteria, virus, radioactive substances, electricity, temperatures and credit payments, and computing investments.

Any quantity that grows or decays by a fixed percent at regular intervals is said to possess **exponential growth** or **exponential decay.**

Exponential Decay

$$f(x) = a \cdot b^x$$

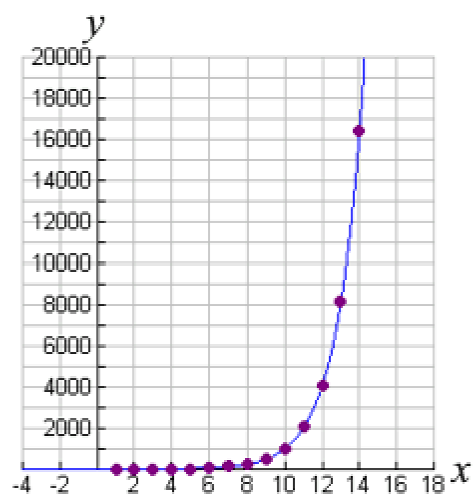
when $a > 0$ and the ***b* is between 0 and 1**, the graph will be decreasing (decaying).



Exponential Growth

$$f(x) = a \cdot b^x$$

when $a > 0$ and the ***b* is greater than 1**, the graph will be increasing (growing).



What is Compound Interest?

http://www.investopedia.com/video/play/what-is-compound-interest/

Compound Interest – Financial Exponential Growth Functions

1. **Monthly, quarterly, semi-annually, annually**

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A(t)$ = Amount of money generated
P = Amount of money P dollars initially invested
r = Annual interest rate (in decimal form)
t = Time (in years)
n = Number of times a year the money is compounded

Compounding	*n*
monthly	12
quarterly	4
semi-annually	2
annually	1

2. **Compounding continuously**

$$A(t) = Pe^{rt}$$

$A(t)$ = Amount of money generated
P = Amount of money P dollars initially invested
r = Annual interest rate (in decimal form)
t = Time (in years)

Example 1: You deposit $2000 to an account that pays 2% annual interest for 4 years. How much does the account earn Monthly? Semi-Annually? Continuously?

Monthly	Semi-Annually	Continuously

Example 2: Suppose that you have $3000 to invest. Which investment yields the greater return over 8 years: 1.2% compounded monthly or 1.3% compounded quarterly?

1.2% Monthly	**1.3% Quarterly**

Example 3: Find the amount of time it would take to get $5000 on a $3000 investment if it is invested at 6% compounded quarterly.

Exponential Growth and Decay Models

The mathematical model for exponential growth or decay problems is represented by

$$A = A_0 e^{kt}$$

where A_0 is the original amount and A is the amount after time t. If $k > 0$, we have exponential growth, and if $k < 0$, we have exponential decay.

Example 4: E. coli bacteria double in population every thirty minutes. If the initial population is 85, what is the population of bacteria after three hours?

Example 5: In 1990 the population of Europe was 509 million and by 2000 it has grown to 729 million. Determine the population in Europe by the year 2010?

Example 6: A scientist started with a culture of 20 bacteria in a dish. The number of bacteria at the end of each successive hour increased exponentially, so that the number at the end of one day was 220. To the nearest million, how many bacteria were there after three days?

Half-Life

Exponential functions are widely used in calculating of half-life of a substance (the amount of time it takes for a substance to **decrease by half**).

$$A = A_0 e^{kt}$$

$$0.5 = e^{kt}$$

Example 7: The half-life of carbon-14 is 100 days. If 400 grams of carbon-14 were left alone. How much carbon will you have left after 65 days?

Example 8: The half-life of radium is 1690 years. If 10 grams are present now, how much will be present in 50 years?