

Inverse Functions and Graphs of Exponential Functions
Module 2, Unit 5, Lesson 9

Background

A function and its inverse function can be described as the "DO" and the "UNDO" functions. A function takes a starting value, performs some operation on this value, and creates an output answer. The inverse function takes the output answer, performs some operation on it, and arrives back at the original function's starting value.

Inverse Function Definition

The relation formed when the independent variable is exchanged with the dependent variable in a given relation.

The **inverse of the function** f and is denoted f^{-1} (read "f-inverse"). The domain of f is equal to the range of f^{-1} , and vice versa.

Finding the Inverse of a Function

1. Replace $f(x)$ with y
2. Interchange x and y
3. Solve for y .
 - a. If this equation does not define y as a function of x , the function f does not have an inverse function and this procedure ends.
 - b. If this equation does define y as a function of x , the function f does have an inverse function.
4. If f has an inverse function, replace y in step 3 by $f^{-1}(x)$. We can verify our results by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Example 1: Find the inverse of the following functions:

a. $f(x) = 7x - 5$

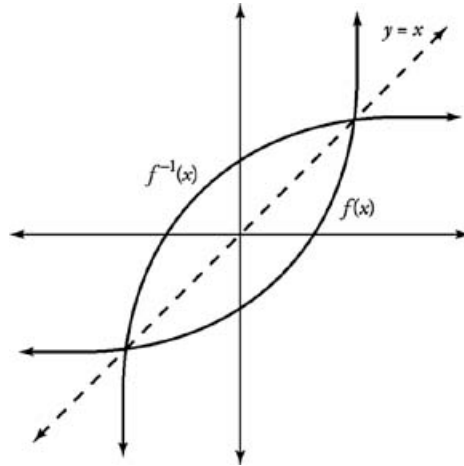
b. $f(x) = x^3 + 1$

c. $f(x) = e^{2x-1}$

Graphs of f and f^{-1}

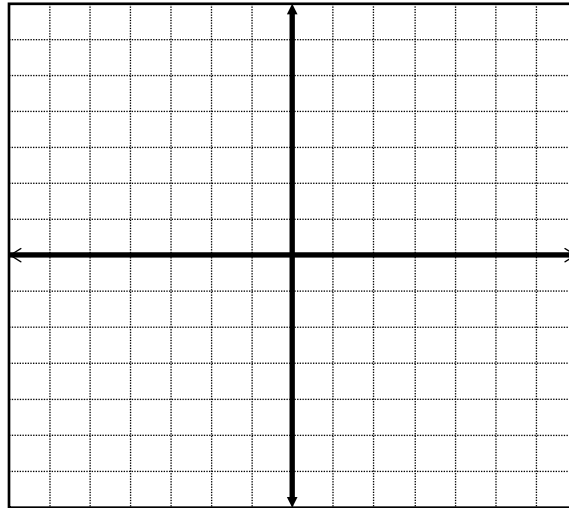
If (a, b) is on the graph of f then the point (b, a) is on the graph of f^{-1} . The points (a, b) and (b, a) are symmetric with respect to the line $y = x$.

The graph of f^{-1} is a reflection of the graph of f about the line $y = x$.



Example 2: Graph f as a solid line and f^{-1} as a dashed line.

$$f(x) = 4x - 2$$



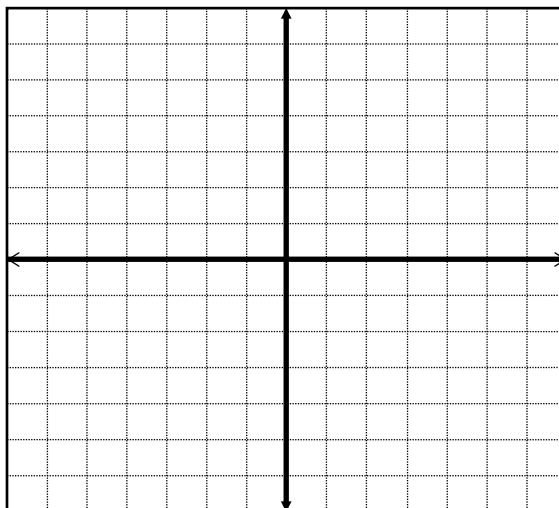
Domain of $f(x) =$ _____

Domain of $f^{-1}(x) =$ _____

Range of $f(x) =$ _____

Range of $f^{-1}(x) =$ _____

Example 3: Find the inverse of $f(x) = x^2 - 1$ if $x \geq 0$. Graph f and f^{-1} .



Domain of $f(x) =$ _____

Domain of $f^{-1}(x) =$ _____

Range of $f(x) =$ _____

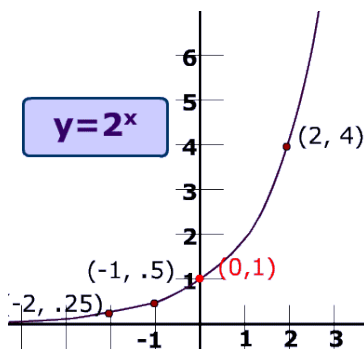
Range of $f^{-1}(x) =$ _____

Exponential Functions

The exponential function f with base b is defined by

$$f(x) = b^x \text{ or } y = b^x$$

where b is a positive constant other than 1 ($b > 0$ and $b \neq 1$) and x is any real number.

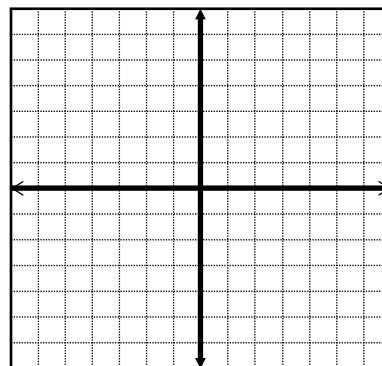


Characteristics of Exponential Function

$$f(x) = b^x$$

1. Domain: $(-\infty, \infty)$
2. Range: $(0, \infty)$
3. Y-intercept: $(0, 1)$
4. Graph rises: $b > 1$
5. Graph falls: $0 < b < 1$
6. Horizontal asymptote: $y = 0$

Example 4: Graph $f(x) = 3^x$



Asymptote: _____

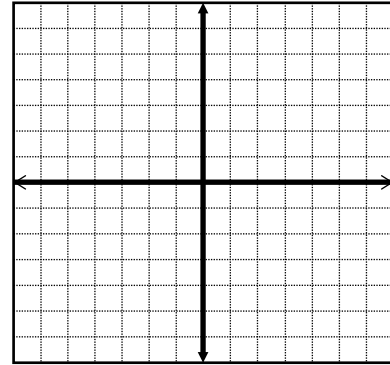
Domain: _____

Range: _____

Transformations of Exponential Functions

Transformation	Graph	Effects on graph
Vertical Shifts	$f(x) = b^x + c$ $f(x) = b^x - c$	Graph shifts up c units Graph shifts down c units c is the horizontal asymptote
Horizontal Shifts	$f(x) = b^{x+c}$ $f(x) = b^{x-c}$	Graph shifts to the left c units Graph shifts to the right c units
Reflection	$f(x) = -b^x$ $f(x) = b^{-x}$	Graph reflected about the x -axis (multiply y by -1) Graph reflected about the y -axis (divide x by -1)
Vertical Stretching or Shrinking	$f(x) = cb^x$	Graph stretches vertically if $c > 1$ and shrinks vertically if $0 < c < 1$. (multiply y by c)
Horizontal Stretching or Shrinking	$f(x) = b^{cx}$	Graph stretches horizontally if $0 < c < 1$. and shrinks vertically if $c > 1$. (divide x by c)

Example 5: Graph $g(x) = 3^{x+1}$

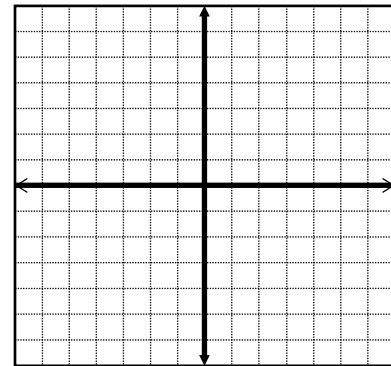


Asymptote: _____

Domain: _____

Range: _____

Example 6: Graph $g(x) = 2^x - 3$



Asymptote: _____

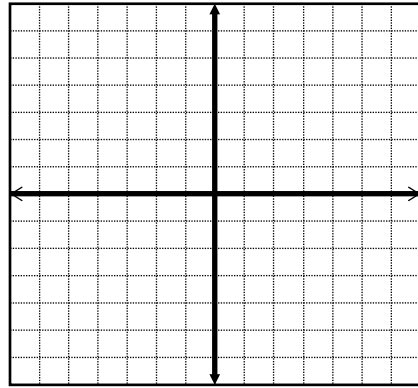
Domain: _____

Range: _____

The Natural Base e

The number e is defined as the value that $\left(1 + \frac{1}{n}\right)^n$ gets closer to as n gets larger and larger. As $n \rightarrow \infty$, then $\left(1 + \frac{1}{n}\right)^n \rightarrow e$. The value of $e \approx 2.718281827\dots$

Example 7: Graph $f(x) = e^x$

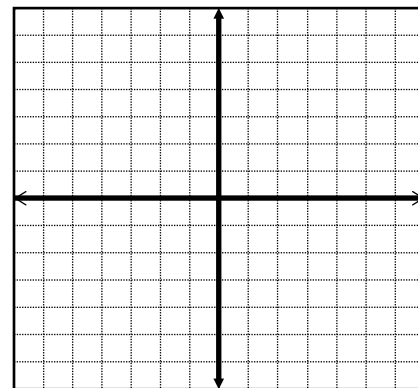


Asymptote: _____

Domain: _____

Range: _____

Example 8: Graph $g(x) = e^{-x} - 1$

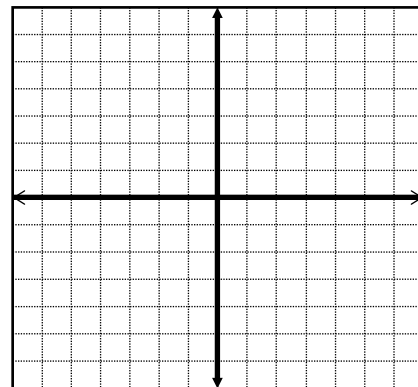


Asymptote: _____

Domain: _____

Range: _____

Example 9: Graph $g(x) = \frac{1}{2}e^{x-3} + 1$



Asymptote: _____

Domain: _____

Range: _____