

Angles on a Unit Circle
Module 3, Unit 6, Lesson 2

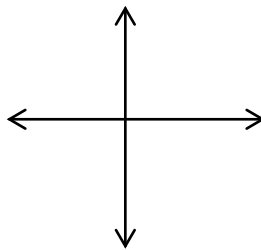
Angles

An *angle* is formed by **two rays** with common endpoints. One side is called the **initial side** while the other is called the **terminal side**.

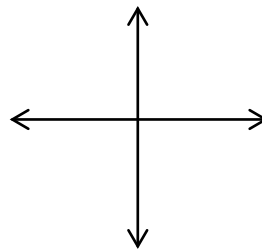
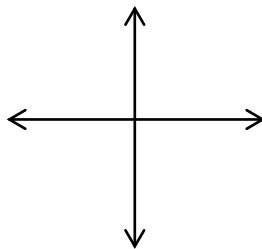
Standard Position

An angle is said to be in **standard position** if

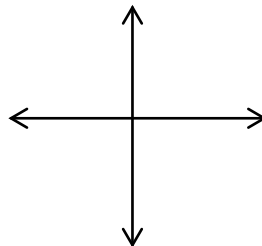
1. The vertex is at the origin of a rectangular coordinate system.
2. The initial side lies along the positive x-axis.



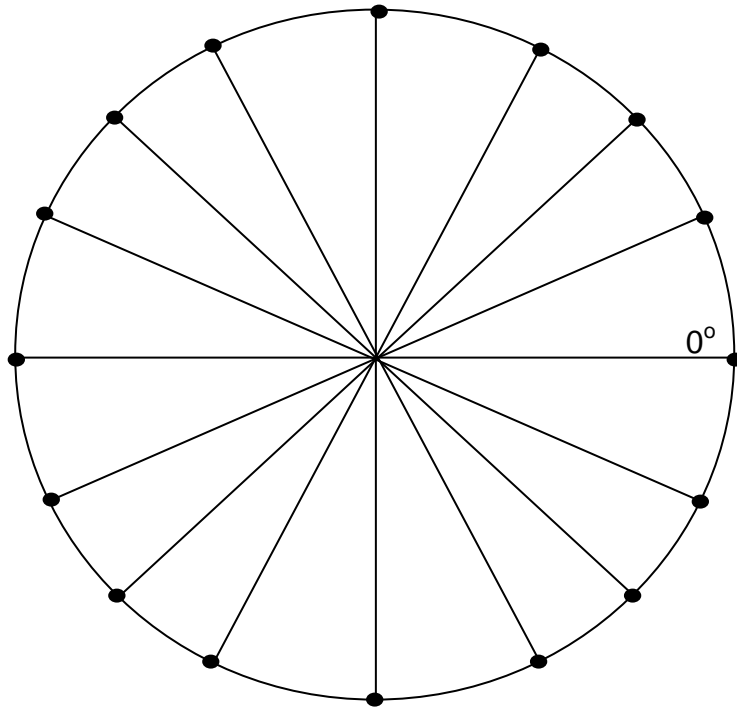
Angles formed by **counterclockwise** rotation are **positive** angles while those formed by **clockwise** rotation are **negative** angles.

**Quadrantal Angles**

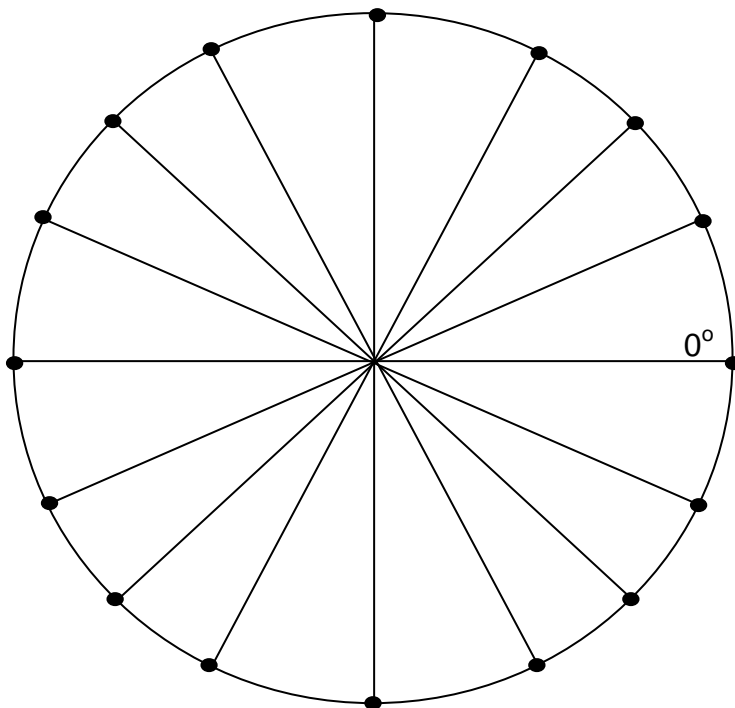
An angle is a **quadrantal angle** if its terminal side lies on the x-axis or the y-axis.



Positive Angles

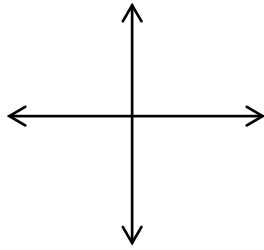


Negative Angles

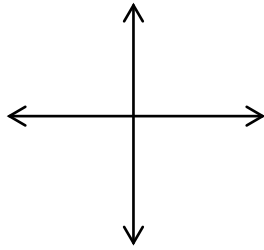


Example 1: Draw and label each angle in standard position.

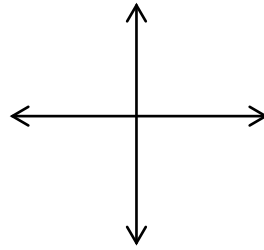
a. 135°



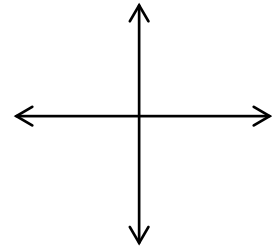
b. 225°



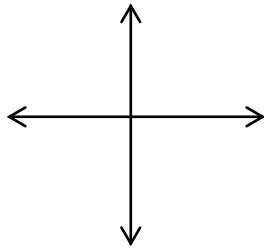
c. -60°



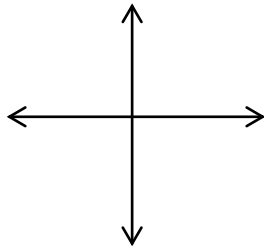
d. 330°



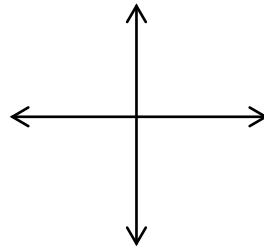
e. -270°



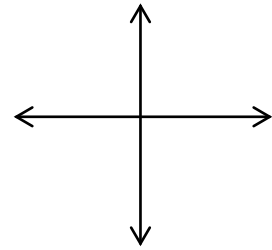
f. 315°



g. 120°

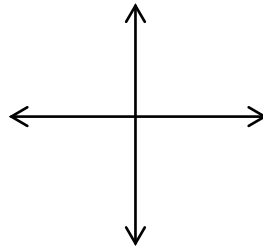


h. -225°



Coterminal Angles

When the measure of an angle in standard position is increased or decreased by multiples of 360° results in **coterminal angles**. This means that an angle θ° is coterminal with angles $\theta^\circ \pm 360^\circ k$, where k is an integer.



Example 2: Find a positive angle less than 360° that is coterminal with the given angles:

a. -135°

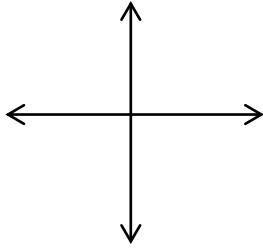
b. 400°

c. 900°

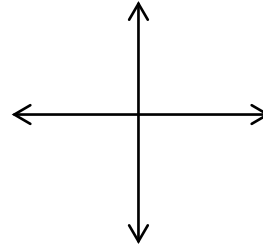
d. -510°

Example 3: Draw and label each angle in standard position.

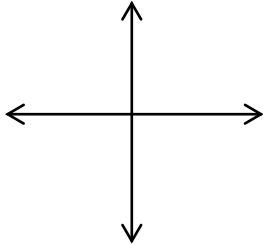
a. 515°



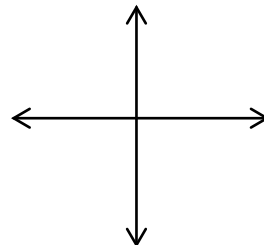
b. -2000°



c. 810°

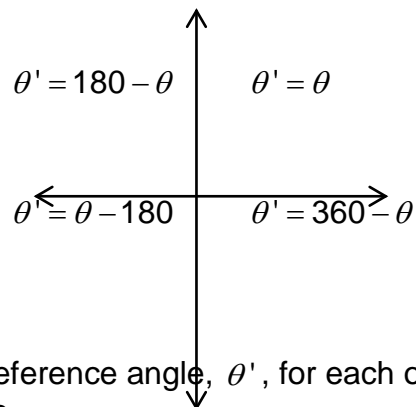


d. -420°



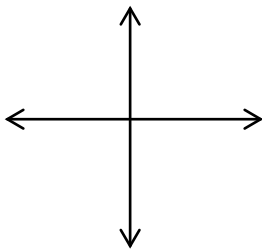
Reference Angles

Given θ in standard position, its reference angle θ' is the smallest positive angle formed by the terminal side of θ and the x-axis.

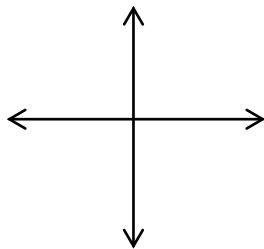


Example 4: Find the reference angle, θ' , for each of the following. Sketch the angle and the reference angle.

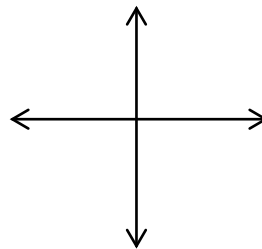
a. 150°



b. 240°



c. -45°



d. 300°

