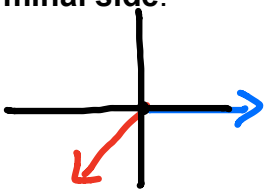
6.2 Notes Name _____

Angles on a Unit Circle

Module 3, Unit 6, Lesson 2

Angles

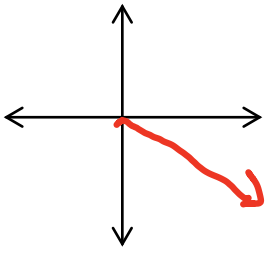
An *angle* is formed by **two rays** with common endpoints. One side is called the **initial side** while the other is called the **terminal side**.



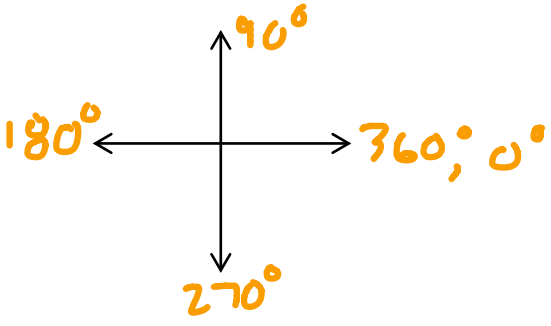
Standard Position

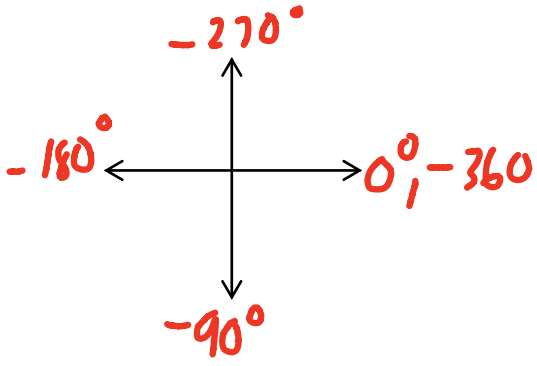
An angle is said to be in **standard position** if

1. The vertex is at the origin of a rectangular coordinate system.
2. The initial side lies along the positive *x*-axis.



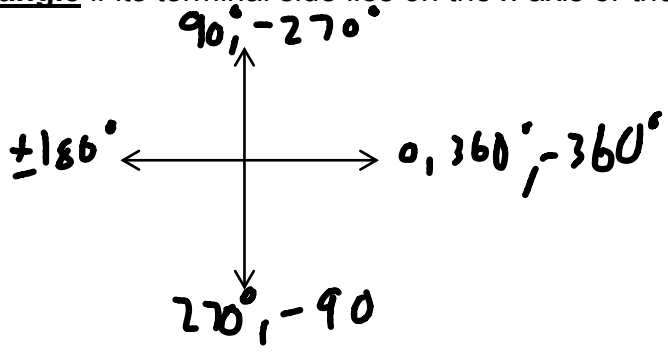
Angles formed by **counterclockwise** rotation are **positive** angles while those formed by **clockwise** rotation are **negative** angles.



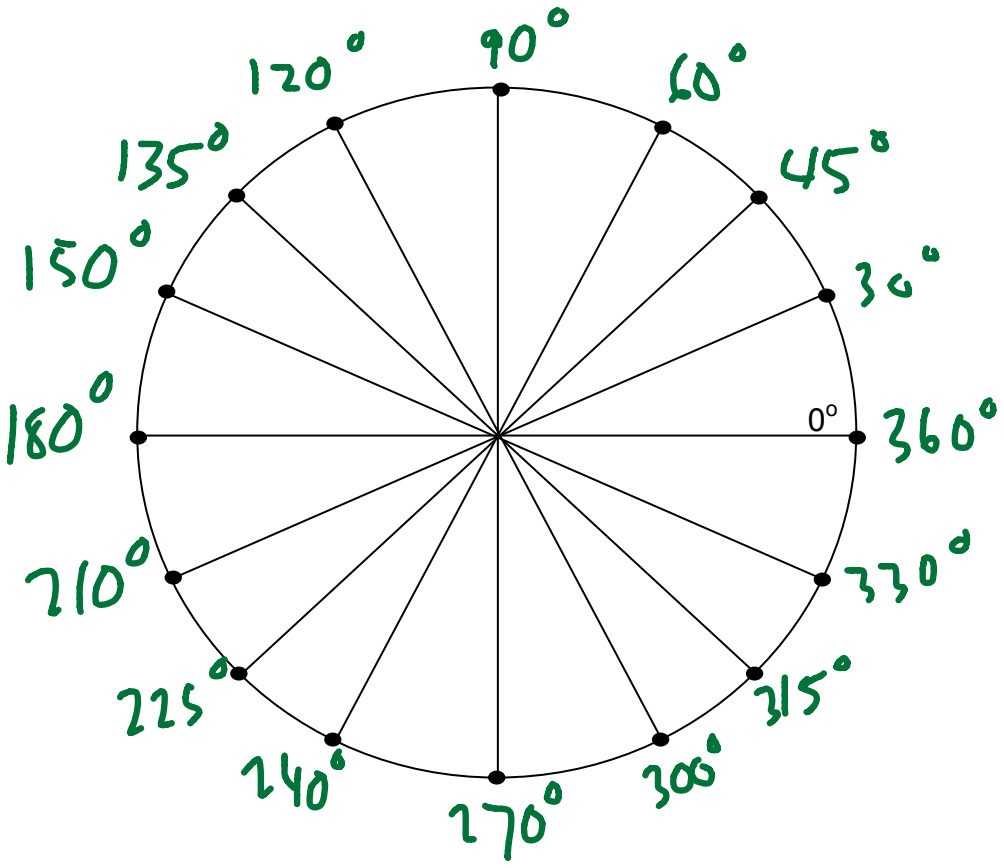


Quadrantal Angles

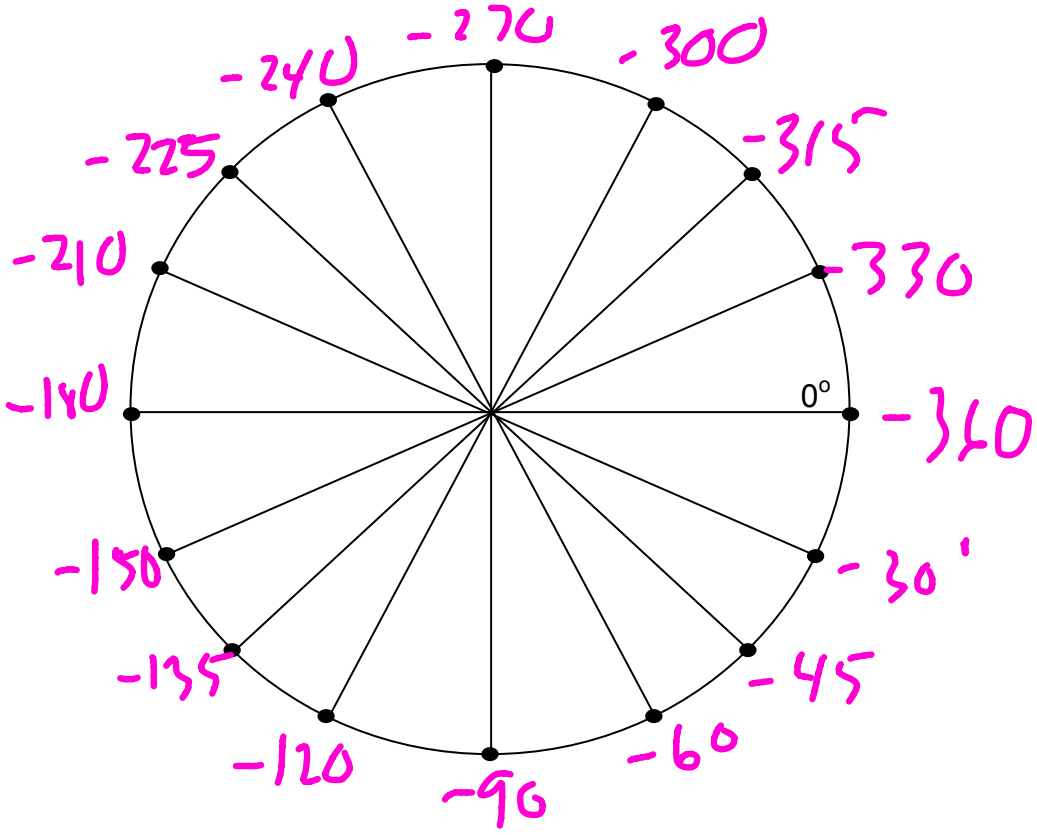
An angle is a **quadrantal angle** if its terminal side lies on the *x*-axis or the *y*-axis.



Positive Angles

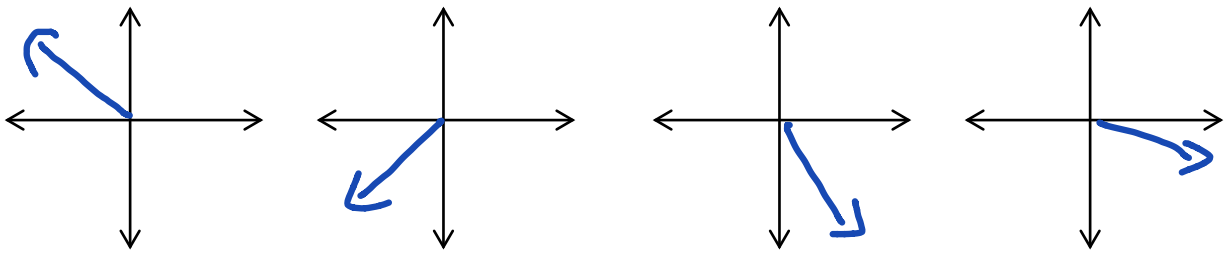


Negative Angles

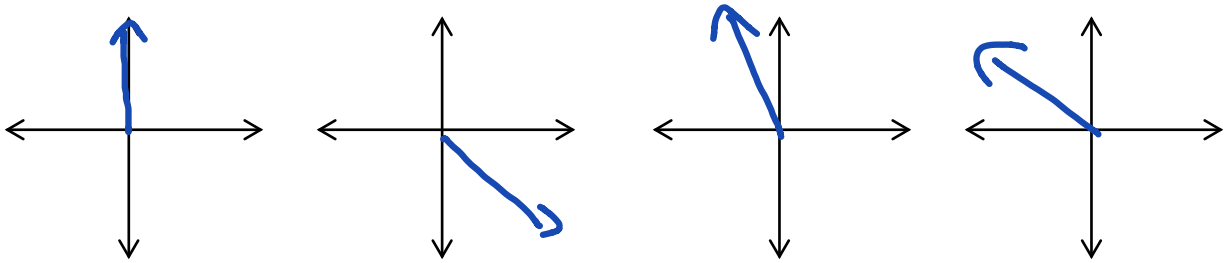


Example 1: Draw and label each angle in standard position.

a. 135° b. 225° c. -60° d. 330°



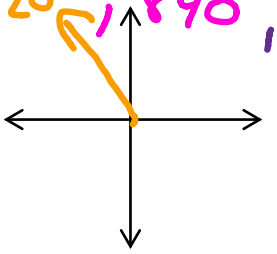
e. -270° f. 315° g. 120° h. -225°



Coterminal Angles

When the measure of an angle in standard position is increased or decreased by multiples of 360° results in **coterminal angles.** This means that an angle θ° is coterminal with angles $\theta^\circ \pm 360^\circ k$, where k is an integer.

$480^\circ, -240^\circ, 120^\circ, 840^\circ, 19{,}560^\circ, -600^\circ$



Example 2: Find a positive angle less than 360° that is coterminal with the given angles:

a. -135° b. 400° c. 900° d. -510°

225°

$-135^\circ + 360 =$

$400^\circ - 360^\circ =$

40°

$900^\circ - 360^\circ =$

$540^\circ - 360^\circ =$

180°

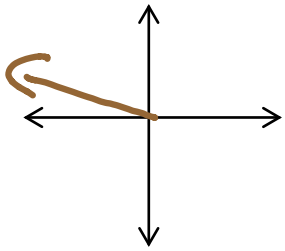
$-510^\circ + 360^\circ =$

$-150^\circ + 360$

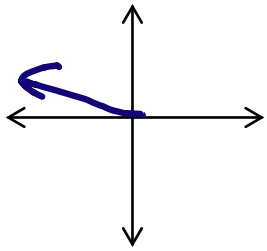
210°

Example 3: Draw and label each angle in standard position.

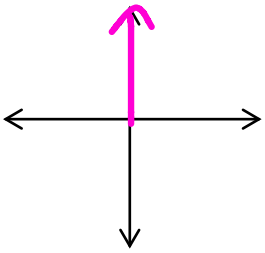
a. 515° $-360 = 155^\circ$



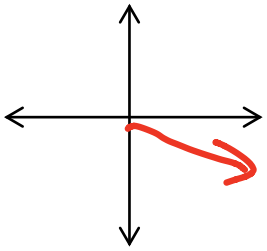
b. -2000° $+2160^\circ = 160^\circ$



c. 810° $-720^\circ = 90^\circ$

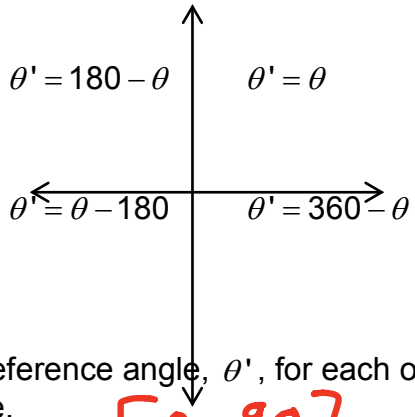


d. -420° $+720^\circ = 300^\circ$



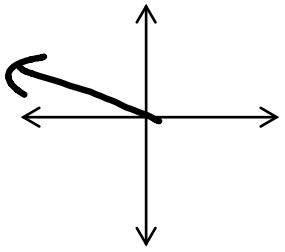
Reference Angles

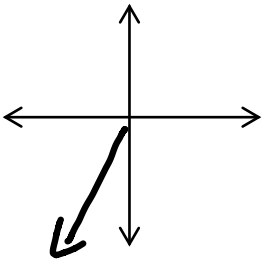
Given θ in standard position, its reference angle θ' is the smallest positive angle formed by the terminal side of θ and the **x-axis**.

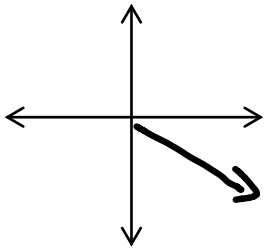


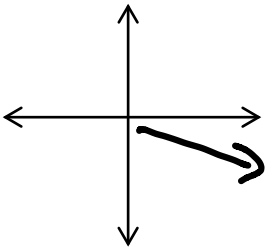
Example 4: Find the reference angle, θ', for each of the following. Sketch the angle and the reference angle.

a. 150° b. 240° $[0, 90]$ c. -45° d. 300°









$180^\circ - 150^\circ =$ 30°

$240^\circ - 180^\circ =$ 60°

$0 - (-45^\circ) =$ 45°

$360^\circ - 300^\circ =$ 60°