

**Finding the Trigonometric Function of Any Angle – Day 2**  
**Module 3, Unit 6, Lesson 5**

**Trigonometric Functions of Any Angle**

Let  $\theta$  be any angle in standard position and let  $P = (x, y)$  be a point on the terminal side of  $\theta$ . If  $r = \sqrt{x^2 + y^2}$  is the distance from  $(0, 0)$  to a point on the circle. Then the trig functions of any angle are:

$$\sin = \frac{y}{r}$$

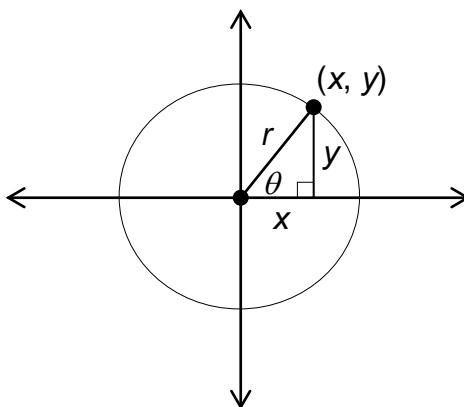
$$\csc = \frac{r}{y}$$

$$\cos = \frac{x}{r}$$

$$\sec = \frac{r}{x}$$

$$\tan = \frac{y}{x}$$

$$\cot = \frac{x}{y}$$



**Finding Trigonometric Functions Given a Point**

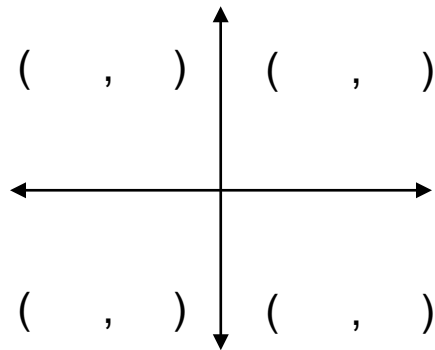
1. Plot the point.
2. Connect the point back to the origin.
3. Connect the point to the x-axis.
4. Label the directional distances.
5. Use Pythagorean Theorem to find the missing hypotenuse.

**Example 1:** Let  $P = (-3, 4)$  be a point on the terminal side of  $\theta$ . Find each of the six trig functions of  $\theta$ .

**Example 2:** Let  $P = (-2, -1)$  be a point on the terminal side of  $\theta$ . Find each of the six trig functions of  $\theta$ .

**Example 3:** Let  $P = (7, -24)$  be a point on the terminal side of  $\theta$ . Find each of the six trig functions of  $\theta$ .

**Where the Trig Functions are Positive**



**Example 4:** If  $\tan \theta < 0$  and  $\cos \theta > 0$ , name the quadrant in which angle  $\theta$  lies.

**Example 5:** If  $\cos \theta < 0$  and  $\sin \theta > 0$ , name the quadrant in which angle  $\theta$  lies.

**Example 6:** If  $\sec \theta < 0$  and  $\csc \theta < 0$ , name the quadrant in which angle  $\theta$  lies.

**Example 7:** Given  $\tan \theta = -\frac{2}{3}$  and  $\sin \theta > 0$ , find the remaining five trigonometric functions.

**Example 8:** Given  $\tan \theta = \frac{5}{12}$  and  $\cos \theta < 0$ , find the remaining five trigonometric functions.

**Example 9:** Given  $\cos \theta = -\frac{3}{5}$  and  $\cot \theta < 0$ , find the remaining five trigonometric functions.

**Example 9:** John was asked to find the  $\sin 210^\circ$ . He followed the following steps.

1. Sketch the angle and found it to be in quadrant III.
2. Found the reference angle by subtraction:  $210^\circ - 180^\circ = 30^\circ$
3. Used his understanding of the  $30^\circ - 60^\circ - 90^\circ$  triangle to find  $\sin 30^\circ = \frac{1}{2}$ .
4. Conclude that since the reference angles for  $210^\circ$  is  $30^\circ$ , then  $\sin 210^\circ = \frac{1}{2}$ .

Do you agree or disagree with John? Explain.