Algebra II/Trig B Name ___________________________

Finding the Trigonometric Function of Any Angle – Day 2
Module 3, Unit 6, Lesson 5

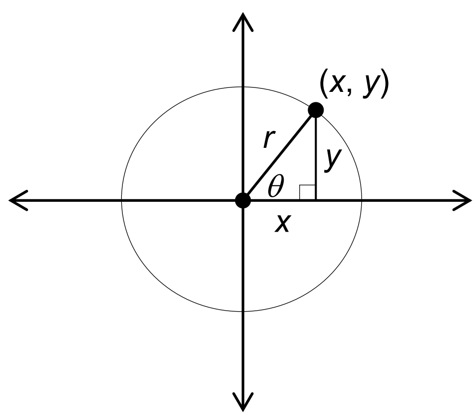
<u>Trigonometric Functions of Any Angle</u>

Let θ be any angle in standard position and let $P=(x,y)$ be a point on the terminal side of θ. If $r=\sqrt{x^2+y^2}$ is the distance from $(0,0)$ to a point on the circle. Then the trig functions of any angle are:

$$\sin=\frac{y}{r} \qquad \csc=\frac{r}{y}$$

$$\cos=\frac{x}{r} \qquad \sec=\frac{r}{x}$$

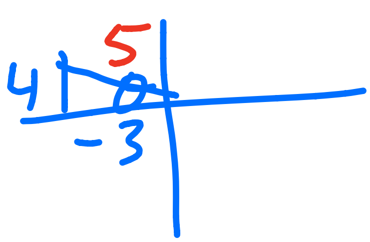
$$\tan=\frac{y}{x} \qquad \cot=\frac{x}{y}$$



<u>Finding Trigonometric Functions Given a Point</u>

1. Plot the point.
2. Connect the point back to the origin.
3. Connect the point to the *x*-axis.
4. Label the directional distances.
5. Use Pythagorean Theorem to find the missing hypotenuse.

<u>Example 1:</u> Let $P=(-3,4)$ be a point on the terminal side of θ. Find each of the six trig functions of θ.

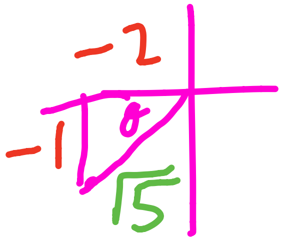


$\sin\theta=\frac{4}{5}$ $\csc\theta=\frac{5}{4}$

$\cos\theta=-\frac{3}{5}$ $\sec\theta=-\frac{5}{3}$

$\tan\theta=-\frac{4}{3}$ $\cot\theta=-\frac{3}{4}$

Example 2: Let $P = (-2, -1)$ be a point on the terminal side of θ. Find each of the six trig functions of θ.

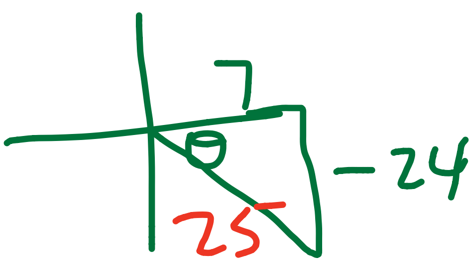


$\sin\theta = -\frac{\sqrt{5}}{5}$ $\csc\theta = -\sqrt{5}$

$\cos\theta = -\frac{2\sqrt{5}}{5}$ $\sec\theta = -\frac{\sqrt{5}}{2}$

$\tan\theta = \frac{1}{2}$ $\cot\theta = 2$

Example 3: Let $P = (7, -24)$ be a point on the terminal side of θ. Find each of the six trig functions of θ.

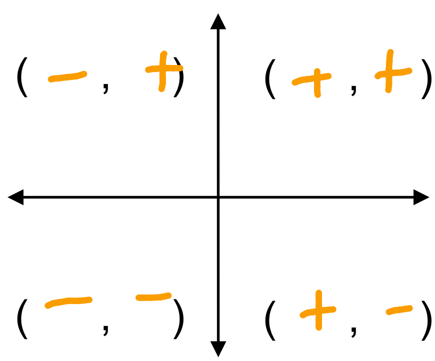


$\sin\theta = -\frac{24}{25}$ $\csc\theta = -\frac{25}{24}$

$\cos\theta = \frac{7}{25}$ $\sec\theta = \frac{25}{7}$

$\tan\theta = -\frac{24}{7}$ $\cot\theta = -\frac{7}{24}$

Where the Trig Functions are Positive



Example 4: If tan θ < 0 and cos θ > 0, name the quadrant in which angle θ lies.

II, IV I, IV

$\boxed{\text{IV}}$

Example 5: If cos θ < 0 and sin θ > 0, name the quadrant in which angle θ lies.

II, III   I, II

$\boxed{II}$

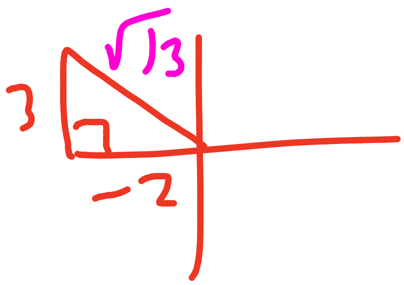
Example 6: If sec θ < 0 and csc θ < 0, name the quadrant in which angle θ lies.

II, III   III, IV

$\boxed{III}$

Example 7: Given $\tan\theta = -\frac{2}{3}$ and sin θ > 0, find the remaining five trigonometric functions.

II, IV   I, II



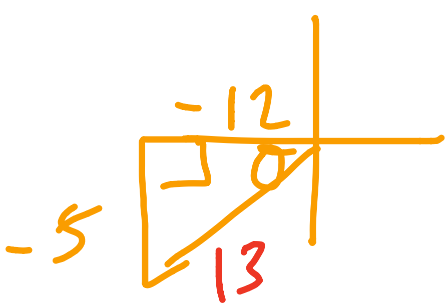
$\sin\theta = \frac{3\sqrt{13}}{13}$ $\csc\theta = \frac{\sqrt{13}}{3}$

$\cos\theta = -\frac{2\sqrt{13}}{13}$ $\sec\theta = -\frac{\sqrt{13}}{2}$

$\cot\theta = -\frac{3}{2}$

Example 8: Given $\tan\theta = \frac{5}{12}$ and cos θ < 0, find the remaining five trigonometric functions.

I, III   II, III

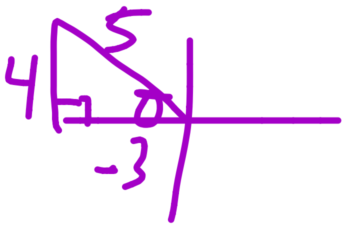


$\sin\theta = -\frac{5}{13}$ $\csc\theta = -\frac{13}{5}$

$\cos\theta = -\frac{12}{13}$ $\sec\theta = -\frac{13}{12}$

$\cot\theta = \frac{12}{5}$

Example 9: Given $\cos\theta = -\frac{3}{5}$ and cot θ < 0, find the remaining five trigonometric functions.



$\sin\theta = \frac{4}{5}$ $\csc\theta = \frac{5}{4}$

$\tan\theta = -\frac{4}{3}$ $\cot\theta = -\frac{3}{4}$

$\sec\theta = -\frac{5}{3}$

Example 9: John was asked to find the sin 210°. He followed the following steps.

1. Sketch the angle and found it to be in quadrant III.
2. Found the reference angle by subtraction: 210° – 180° = 30°
3. Used his understanding of the 30° – 60° – 90° triangle to find sin 30° = $\frac{1}{2}$.
4. Conclude that since the reference angles for 210° is 30°, then sin 210° = $\frac{1}{2}$.

Do you agree or disagree with John? Explain.