

Inverse Trigonometric Functions  
Module 3, Unit 7, Lesson 6

**The Inverse Sine Function**

The inverse sine function, denoted by  $\sin^{-1}$ , is the inverse of the restricted sine function

$$y = \sin x, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}. \quad \text{Thus,}$$

$$y = \sin^{-1} x \quad \text{means} \quad \sin y = x$$

<b>NOTE:</b> $y = \sin^{-1} x$ $\downarrow$ Inverse sine function	<b><u>does not mean</u></b>	$y = (\sin x)^{-1}$ $\downarrow$ Reciprocal of sine function
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**Finding Exact Values of  $\sin^{-1} x$**

1. Let  $\theta = \sin^{-1} x$
2. Rewrite  $\theta = \sin^{-1} x$  as  $\sin \theta = x$ , where  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
3. Find the value of  $\theta$  in  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\theta$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin \theta$									

**Example 1:**

Find the exact value of

- |                                   |                                   |  |
|-----------------------------------|-----------------------------------|--|
| a. $\sin^{-1} \frac{\sqrt{2}}{2}$ | b. $\sin^{-1} \frac{\sqrt{3}}{2}$ | c. $\sin^{-1} \left(-\frac{1}{2}\right)$ |
|-----------------------------------|-----------------------------------|--|

**The Inverse Cosine Function**

The inverse cosine function, denoted by  $\cos^{-1}$ , is the inverse of the restricted cosine function  $y = \cos x, 0 \leq x \leq \pi$ . Thus,

$$y = \cos^{-1} x \quad \text{means} \quad \cos y = x$$

### Finding Exact Values of $\cos^{-1} x$

1. Let  $\theta = \cos^{-1} x$
2. Rewrite  $\theta = \cos^{-1} x$  as  $\cos \theta = x$ , where  $0 \leq \theta \leq \pi$
3. Find the value of  $\theta$  in  $[0, \pi]$

$\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$\cos \theta$									

### Example 2:

Find the exact value of

a.  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

b.  $\cos^{-1}\left(-\frac{1}{2}\right)$

c.  $\cos^{-1}1$

### The Inverse Tangent Function

The inverse tangent function, denoted by  $\tan^{-1}$ , is the inverse of the restricted tangent function  $y = \tan x$ ,  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ . Thus,

$$y = \tan^{-1} x \text{ means } \tan y = x$$

### Finding Exact Values of $\tan^{-1} x$

1. Let  $\theta = \tan^{-1} x$
2. Rewrite  $\theta = \tan^{-1} x$  as  $\tan \theta = x$ , where  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
3. Find the value of  $\theta$  in  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\theta$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan \theta$									

### Example 3:

Find the exact value of

a.  $\tan^{-1}\sqrt{3}$

b.  $\tan^{-1}1$

c.  $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

## Inverse Properties

### The Sine Function and Its Inverse

$$\sin(\sin^{-1} x) = x \quad \text{for every } x \text{ in the interval } [-1, 1]$$

$$\sin^{-1}(\sin x) = x \quad \text{for every } x \text{ in the interval } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

### The Cosine Function and Its Inverse

$$\cos(\cos^{-1} x) = x \quad \text{for every } x \text{ in the interval } [-1, 1]$$

$$\cos^{-1}(\cos x) = x \quad \text{for every } x \text{ in the interval } [0, \pi]$$

### The Tangent Function and Its Inverse

$$\tan(\tan^{-1} x) = x \quad \text{for every real number } x$$

$$\tan^{-1}(\tan x) = x \quad \text{for every } x \text{ in the interval } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

### Example 4:

Find the exact value, if possible:

1.  $\sin(\sin^{-1} 4.5) =$

2.  $\sin^{-1}\left(\sin \frac{7\pi}{6}\right) =$

3.  $\cos^{-1}\left(\cos \frac{3\pi}{4}\right) =$

4.  $\tan(\tan^{-1} 7) =$

5.  $\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right) =$

6.  $\cos^{-1}\left(\cos \frac{5\pi}{4}\right) =$

**Example 5:**

Find the exact value.

a.  $\cos\left(\tan^{-1}\frac{5}{12}\right)$

b.  $\sin\left(\tan^{-1}\frac{3}{4}\right)$

c.  $\cot\left[\sin^{-1}\left(-\frac{1}{3}\right)\right]$

**Practice**

Find the exact value of each expression.

1.  $\sin^{-1}1$

2.  $\cos^{-1}1$

3.  $\tan^{-1}1$

4.  $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

5.  $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

6.  $\sin(\cos^{-1}0)$

7.  $\csc\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$

8.  $\tan\left[\sin^{-1}\left(-\frac{3}{5}\right)\right]$

9.  $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$