Algebra II/Trig B

Module 4, Unit 8, Lessons 3 & 4 : Conditional Probability and Independence

Warm-up- Rufus King High School

Students at Rufus King High School were discussing some of the challenges of finding space for athletic teams to practice after school. Part of the problem, according to Kristin, is that female students are more likely to be involved in an after-school athletics program than male students. However, the athletic director assigns the available facilities as if male students are more likely to be involved. Before suggesting changes to the assignments, the students decided to investigate.

Suppose the following information is known about Rufus King High School: 40% of students are involved in one or more of the after-school athletics programs offered at the school. It is also known that 58% of the school's students are female. The students decide to construct a hypothetical 1000 two-way table, like Table 1, to organize the data.

	Yes—Participate in After-School Athletics Programs	**No—Do Not Participate in After-School Athletics Programs**	**Total**
Female	Cell 1	Cell 2	Cell 3
Male	Cell 4	Cell 5	Cell 6
Total	Cell 7	Cell 8	Cell 9

1. Based on the given information, which cells are you able to fill in for the table? Fill them in.

2. Which cells are you not able to fill in? Why are you NOT able to fill them in?

Definitions and Notation

Individual events are often represented with capital letters for ease.

$\boldsymbol{A}$ **= a randomly selected student is female.**

$\boldsymbol{B}$**= a randomly selected student participates in an after-school athletics program.**

Two events are complementary if:

they do not contain any outcomes in common AND together contain all possible outcomes.

If "A" represents event A, the complement of A is represented as $\boldsymbol{A^C}$.

The event $\boldsymbol{A^C}$ ("not A") represents the event "a randomly selected student is not female."

What does $\boldsymbol{B^C}$ ("not B") represent?

The notation $\boldsymbol{A} \cup \boldsymbol{B}$ represents event "A or B." In this case it represents: the event "a randomly selected student is female or participates in an after school athletics program." (Beware of double-counting.)

The notation $\boldsymbol{A} \cap \boldsymbol{B}$ represents event "A and B." What does it represent here?

The athletic director indicated that 23.2% of the students at Rufus King are female and participate in after-school athletics programs. Based on this information, and your warm-up, complete the table below.

	Yes—Participate in After-School Athletic Program	***No—Do Not Participate in After-School Athletic Program***	***Total***
Female			
Male			
Total			

Based on the descriptions and the above table, determine the probability of each of the following events:

a. $P(A)$

b. P(B)

c. P(A^C)

d. $P(B^C)$

e. $P(A \cup B)$

f. $A \cap B$

Determine the following values:

a. $P(A) + P(A^C)$

b. $P(B) + P(B^C)$

What do you notice about the results of parts (a) and (b)? Explain.

Conditional Probability

Another type of probability is called a *conditional probability*. Pulling apart the two-way table helps to define a conditional probability.

	Yes—Participate in After-School Athletics Program	No—Do Not Participate in After-School Athletics Program	Total
Female	Cell 1	Cell 2	Cell 3

Suppose that a randomly selected student is female. What is the probability that the selected student participates in an after-school athletics program? This probability is an example of what is called a *conditional probability*. This probability is calculated as the number of students who are female and participate in an after-school athletics program (or the students in cell 1) divided by the total number of female students (or the students in cell 3).

The notation for conditional probability is $P(A|B)$,

which is the probability of event A happening GIVEN event A has already happened.

The following are also examples of conditional probabilities. Provide the appropriate notation and answer each question.

a. What is the probability that a randomly selected female participates in the after-school athletic program?

b. What is the probability that a randomly selected female does not participate in after-school athletics?

c. What is the probability that a randomly selected male participates in an after-school athletics program?

d. A student is selected at random. What is the probability this student participates in an after-school athletics program?

Based on the above answers to do you think that female students are more likely to be involved in after-school athletics programs? Explain your answer.

What might explain the concern female students expressed in the beginning of this lesson about the problem of assigning practice space?

Independence: ***Students REALLY struggle with this idea and how to prove it.***

In the example above, conditional probabilities were used to investigate whether or not there is a connection between two events.

Defintion: Two events are independent when knowing that one event has occurred **does not change** the likelihood that the second event has occurred.

Notation: A and B are independent events IF $P(A|B) = P(A)$ or $P(B|A) = P(B)$

Pairs of Events that ARE Independent

Pairs of Events that are NOT Independent

How can conditional probabilities be used to tell if two events are independent or not independent?

Recall the hypothetical 1000 two-way frequency table that was used earlier to classify students at Rufus King High School according to gender and whether or not they participated in an after-school athletics program.

Use the table to calculate each of the probabilities described below.

e. The probability that a randomly selected student participates in an after-school athletics program

f. The probability that a randomly selected student who is female participates in an after-school athletics program

g. The probability that a randomly selected student who is male participates in an after-school athletics program

Would your prediction of whether or not a student participates in an after-school athletics program change if you knew the gender of the student? Explain your answer.

Two events are ***independent*** if knowing that one event has occurred does not change the probability that the other event has occurred. For example, consider the following two events:

F: The event that a randomly selected student is female

S: The event that a randomly selected student participates in an after-school athletics program

Events F and S would be independent if: $\boldsymbol{P(S) = P(S|F)}$ **and/or** $\boldsymbol{P(F) = P(F|S)}$

Only one of the above needs to be proven to show independence

1) the probability that a randomly selected student participates in an after-school athletics program

   is equal to

   the probability that a randomly selected student who is female participates in an after-school athletics program.

$$\boldsymbol{P(S) = P(S|F)}$$

OR

2) the probability that a randomly selected student is female

   is equal to

   the probability that a randomly selected student who participates in an after-school athletics program is female.

$$\boldsymbol{P(F) = P(F|S)}$$

If this was the case, knowing that a randomly selected student is female does not change the probability that the selected student participates in an after-school athletics program. In addition, knowing that a randomly selected student participates in an after-school activity does not change the probability that the student is female.

Then, $\boldsymbol{F}$ and $\boldsymbol{S}$ would be independent.

Based on the definition of independence, are the events “randomly selected student is female” and “randomly selected student participates in an after-school athletics program” independent? What two ways could we test it?

Example Asthma and Smoking: Consider the data below.

Event A = student has asthma; Event B = at least one household member smokes

	No Household Member Smokes	**At Least One Household Member Smokes**	**Total**
Student Has Asthma	69	113	182
Student Does Not Have Asthma	473	282	755
Total	542	395	937

a. You are asked to determine if the two events "a randomly selected student has asthma" and "a randomly selected student has a household member who smokes" are independent. What probabilities could you calculate to answer this question?

b. Calculate the probabilities you described above.

c. Based on the probabilities you calculated, are these two events independent or not independent? Explain.

d. Is the probability that a randomly selected student who has asthma and who has a household member who smokes the same as or different from the probability that a randomly selected student who does not have asthma but does have a household member who smokes? Explain your answer.

e. A student is selected at random. The selected student indicates that he has a household member who smokes. What is the probability that the selected student has asthma?