Module 4, Unit 8, Lesson 6: Probability Rules

Example 1: The Complement Rule

In previous lessons, you have seen that to calculate the probability that an event *does not happen,* you can subtract the probability of the event from 1. If the event is denoted by A, then this rule can be written:

$$P(\text{not } A) = 1 - P(A).$$

Using complement notation, this can also be written as $P(A^C) = 1 - P(A)$

For example, suppose that the probability that a particular flight is on time is 0.78. What is the probability that the flight is not on time?

Example 2: Formula for Conditional Probability

When a room is randomly selected in a downtown hotel, the probability that the room has a king-sized bed is 0.62, the probability that the room has a view of the town square is 0.43, and the probability that it has a king-sized bed *and* a view of the town square is 0.38. Let A be the event that the room has a king-sized bed, and let B be the event that the room has a view of the town square.

a. What is the meaning of $P(A \text{ given } B)$ in this context?

b. Use a hypothetical 1000 table to calculate $P(A \text{ given } B)$.

	A **(room has a king-sized bed)**	**Not *A*** **(room does not have a king-sized bed)**	**Total**
B **(room has a view of the town square)**			
Not *B* **(room does not have a view of the town square)**			
Total			

There is also a formula for calculating a conditional probability. The formula for conditional probability is

$$P(A \text{ given } B) = \frac{P(A \text{ and } B)}{P(B)}$$

c. Use this formula to calculate $P(A \text{ given } B)$, where the events A and B are as defined in this example. How does the probability you calculated using the formula compare to the probability you calculated using the hypothetical 1000 table?

Practice

A credit card company states that 42% of its customers are classified as long-term cardholders, 35% pay their bills in full each month, and 23% are long-term cardholders who also pay their bills in full each month. Let the event that a randomly selected customer is a long-term cardholder be L and the event that a randomly selected customer pays his bill in full each month be F.

a. What are the values of $P(L)$, $P(F)$, and $P(L \text{ and } F)$?

b. Draw a Venn diagram, and label it with the probabilities from part (a).

c. Use the conditional probability formula to calculate $P(L \text{ given } F)$. (Round your answer to the nearest thousandth.)

d. Use the conditional probability formula to calculate $P(F \text{ given } L)$. (Round your answer to the nearest thousandth.)

e. Which is greater, $P(F \text{ given } L)$ or $P(F)$? Explain why this is relevant.

f. Remember that two events A and B are said to be independent if $P(A \text{ given } B) = P(A)$. Are the events F and L independent? Explain.

Multiplication Rule for Independent Events
If events A and B are independent, then the probability of A and B happening is: $P(A \cap B) = P(A) \cdot P(B)$

We can also use this formula as another method to check for independence.
IF $P(A) \cdot P(B) = P(A \cap B)$, then events A and B are independent.

Example 3: Using the Multiplication Rule for Independent Events

A number cube has faces numbered 1 through 6, and a coin has two sides, heads and tails.

The number cube will be rolled, and the coin will be flipped. Find the probability that the cube shows a 4 and the coin lands on heads. Because the events are independent, we can use the multiplication rules we just learned.

If you toss the coin five times, what is the probability you will see a head on all five tosses?

If you tossed the coin five times and got five heads, would you think that this coin is a fair coin? Why or why not?

If you roll the number cube three times, what is the probability that it will show 4 on all three throws?

If you rolled the number cube three times and got a 4 on all three rolls, would you think that this number cube is fair? Why or why not?

Suppose that the credit card company introduced in Practice states that when a customer is selected at random, the probability that the customer pays his bill in full each month is 0.35, the probability that the customer makes regular online purchases is 0.83, and these two events are independent. What is the probability that a randomly selected customer pays his bill in full each month *and* makes regular online purchases?

Exercise 2

A spinner has a pointer, and when the pointer is spun, the probability that it stops in the red section of the spinner is 0.25.

a. If the pointer is spun twice, what is the probability that it will stop in the red section on both occasions?

b. If the pointer is spun four times, what is the probability that it will stop in the red section on all four occasions? (Round your answer to the nearest thousandth.)

c. If the pointer is spun five times, what is the probability that it never stops on red? (Round your answer to the nearest thousandth.)

Practice

1. Of the light bulbs available at a store, 42% are fluorescent, 23% are labeled as long life, and 12% are fluorescent *and* long life.
   a. A light bulb will be selected at random from the light bulbs at this store. Rounding your answer to the nearest thousandth where necessary, find the probability that
      i. The selected light bulb is not fluorescent.

      ii. The selected light bulb is fluorescent given that it is labeled as long life.

   b. Are the events "fluorescent" and "long life" independent? Explain. Use ALL three methods referenced at the bottom to check. (On the homework you only need to do one of them.)

2. When a person is selected at random from a very large population, the probability that the selected person is right-handed is 0.82. If three people are selected at random, what is the probability that
   a. They are all right-handed?

   b. None of them is right-handed?

Lesson Summary

For any event A, $P(\text{not } A) = 1 - P(A)$.

For any two events A and B, $P(A \text{ given } B) = \frac{P(A \text{ and } B)}{P(A)}$.

Events A and B are independent if and only if $P(A \text{ and } B) = P(A)P(B)$.

Checking for independence: events A and B can be considered to be independent if we can show:

1. $P(A) = P(A|B)$ OR
2. $P(B) = P(B|A)$ OR
3. $P(A \text{ and } B) = P(A) \cdot P(B)$