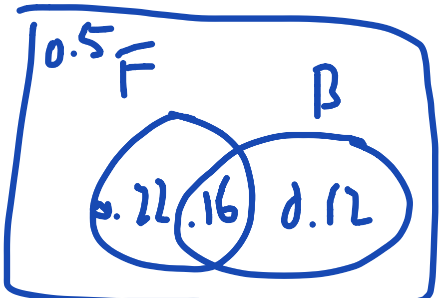
Module 4, Unit 8, Lesson 7: Applications of Probability Rules

Warm-up

When a car is brought to a repair shop for a service, the probability that it will need the transmission fluid replaced is 0.38, the probability that it will need the brake pads replaced is 0.28, and the probability that it will need both the transmission fluid and the brake pads replaced is 0.16. Let the event that a car needs the transmission fluid replaced be F T and the event that a car needs the brake pads replaced be B.

a. What are the values of the following probabilities? (Hint: Use a Venn Diagram or hypothetical 1,000 table)

$P(F) = 0.38$ $\quad P(B) = 0.28$ $\quad P(F \text{ and } B)$ 0.16



	B	B^c	T
F	160	220	380
F^c	120	500	620
T	280	720	1000

b. Find the probability that a randomly selected car needs the transmission fluid or the brake pads replaced.

$P(F \cup B) = 0.38 + 0.28 - 0.16 = \boxed{0.5}$

The Addition Rule
For any two events A and B, $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
Using probability notation: $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Example 1: Multiplication and Addition Rules

Josie will soon be taking exams in math (M) and Spanish (S). She estimates that the probability she passes the math exam is 0.9, and the probability that she passes the Spanish exam is 0.8. She is also willing to assume that the results of the two exams are independent of each other.

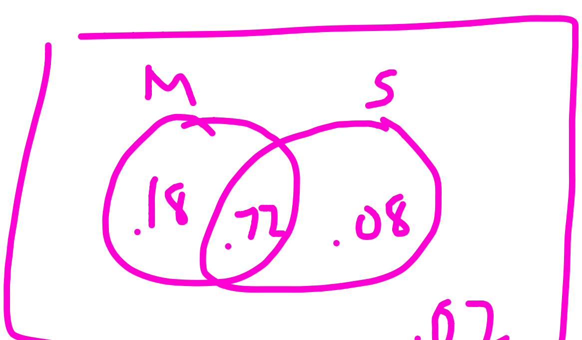
c. Using Josie's assumption of independence, calculate the probability that she passes both exams.

$P(M \cap S) = 0.9(0.8) = \boxed{0.72}$

d. Using the Addition Rule, find the probability that Josie passes at least one of the exams. (Hint: Passing at least one of the exams is passing math *or* passing Spanish.)

$P(M \cup S) = 0.9 + 0.8 - 0.72 = \boxed{.98}$

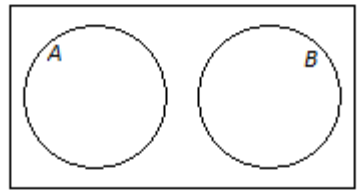
e. How could we represent this situation using a Venn diagram?



Disjoint Events

Two events are said to be *disjoint* if they have no outcomes in common. So, if the events A and B are disjoint, the Venn diagram looks the one to the right.

Disjoint (aka mutually exclusive) events cannot both happen at the same time.



Examples of Disjoint Events ***Examples of Non-Disjoint Events***

- Cards (uncommon values)
- Coin and dice
- Some Sports (Same Season)

If A and B are disjoint, then $P(A \text{ and } B) =$ 0.

So, the addition rule for disjoint events can be written as $P(A \text{ or } B) = P(A) + P(B)$.

Example 2: Use of the Addition Rule for Disjoint Events

A set of 40 cards consists of the following: 10 black cards showing squares, 10 black cards showing circles, 10 red cards showing X's, 10 red cards showing diamonds

A card will be selected at random from the set. Find the probability that the card is black or shows a diamond.

$$\frac{20}{40} + \frac{10}{40} = \frac{30}{40} = \boxed{0.75}$$

Example 3: Combining Use of the Multiplication and Addition Rules

A red cube has faces labeled 1 through 6, and a blue cube has faces labeled in the same way. The two cubes are rolled. Find the probability of each event.

a. Both cubes show 6's.

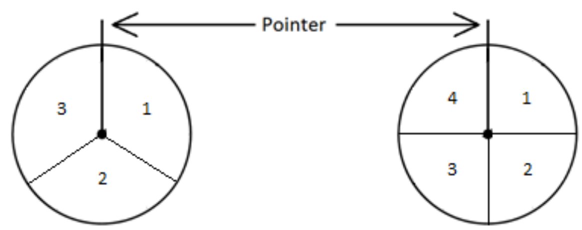
$$\frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36}}$$

b. The total score is at least 11.

$$\left(\frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6}\right) = \boxed{\frac{1}{12}}$$

Example 4

The diagram to the right shows two spinners. For the first spinner, the scores 1, 2, and 3 are equally likely, and for the second spinner, the scores 1, 2, 3, and 4 are equally likely. Both pointers will be spun. Writing your answers as fractions in lowest terms, find the probability of each event.



a. The total of the 2 scores on the spinners is 2.

$$\frac{1}{3} \cdot \frac{1}{4} = \boxed{\frac{1}{12}}$$

b. The total of the 2 scores is 3.

$$\left(\frac{1}{3} \cdot \frac{1}{4}\right) + \left(\frac{1}{3} \cdot \frac{1}{4}\right) = \boxed{\frac{1}{6}}$$

c. The total of the 2 scores is 5.

$$3\left(\frac{1}{12}\right) = \boxed{\frac{1}{4}}$$

d. The total of the 2 scores is not 5.

$$1 - \frac{1}{4} = \boxed{\frac{3}{4}}$$

Example 5

When a call is received at an airline's call center, the probability that it comes from abroad is 0.32, and the probability that it is to make a change to an existing reservation is 0.38.

a. Suppose that you are told that the probability that a call is both from abroad and is to make a change to an existing reservation is 0.15. Calculate the probability that a randomly selected call is either from abroad or is to make a change to an existing reservation.

$P(A \cap B) = 0.15$

$0.32 + 0.38 - 0.15 = 0.7 - 0.15$

$\boxed{0.55}$

b. Suppose now that you are *not* given the information in part (a), but you are told that the events "the call is from abroad" and "the call is to make a change to an existing reservation" are independent. What is the probability that a randomly selected call is either from abroad or is to make a change to an existing reservation?

c. What is the probability that three calls in a row are about changing an existing reservation?

d. What is the probability that there are 5 calls received, and only the 5^{th} call is from abroad?

e. If there are 4 calls received, what is the probability that at least one of them is to make a change to an existing reservation?

f. What is the probability that neither of the next two calls are from abroad nor involve changing an existing reservation?

Example 6

A golfer will play two holes of a course. Suppose that on each hole the player will score 3, 4, 5, 6, or 7, with these five scores being equally likely. Find the probability, and explain how the answer was determined that the player's total score for the two holes will be

a. 14.

b. 12.

Example 7- Multiplication Rule with Independent and Dependent Events

Pay close attention to how these two problems are the same, yet how they are different.

Light Bulbs: Twelve light bulbs are tested to see if they last as long as the manufacturer claims they do. Three light bulbs fail the test. Two light bulbs are selected at random without replacement.

a. Find the probability that both light bulbs failed the test.

b. Find the probability that both light bulbs passed the test.

c. Find the probability that at least one light bulb failed the test.

Guessing: A multiple-choice quiz has three questions. Each with five answer choices. Only one of the choices is correct. You have no idea what the answer is to any question and have to guess each answer.

a. Find the probability of answering the first question correctly.

b. Find the probability of answering the first two questions correctly.

c. Find the probability of answering all three questions correctly.

d. Find the probability of answering none of the questions correctly.

e. Find the probability of answering at least one of the questions correctly.