Module 4, Unit 9 Lesson 10 – Normal Distributions

Warm-up: SATs and ACTs

Suppose that an admissions officer at Wildcat University (WU), Juanita, needs to compare scores of students who take the Scholastic Aptitude Test (SAT) with those who take the American College Test (ACT). Among applicants to WU who take the SAT, the mean is 896 and the standard deviation is 174. Among WU applicants who take the ACT, the mean is 20.6 and the standard deviation is 5.2.

Juanita is considering both Jasmine and Enrique for admission. Enrique scored 1080 on the SAT, and Jasmine scored 28 on the ACT. She wishes to determine whose test score is more impressive.

a) Juanita's assistant tells her she should choose Enrique because his test score is higher. Do you agree with this logic? Why or why not?

b) How many points above the school's SAT mean was Enrique's score?

c) How many points above the school's ACT mean was Jasmine's score?

d) Juanita concludes that, since Enrique's test score is higher AND it is more points above his test's mean, he should be chosen. What is the problem with this conclusion?

e) How could Juanita evaluate the candidates on an even playing field?

Z-scores

When we want to compare values from two different distributions (aka find a measure of relative standing) OR we want to evaluate a value within the context of a given distribution, we often use z-scores. A z-score for a particular value measures the number of standard deviations away from the mean. A positive z-score corresponds to a value that is above the mean, and a negative z-score corresponds to a value that is below the mean. A z-score is calculated by

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}.$$

If a value is at the mean, its z-score will be _____________. The further a z-score is from ______, the more unusual its corresponding value is considered. In most instances, a larger z-score represents a value that is more impressive. We round z-scores to the nearest hundredth

Calculate the z-score for each applicant. Use them to determine whether Juanita should admit Enrique or Jasmine. Justify your answer by referencing the z-scores.

Example 1

Maria earned a 75 on her Chemistry test and an 85 on her Algebra 2/Trig test. The mean and standard deviation for the Chemistry test were 72 and 4. The mean and standard deviation for the Algebra 2/Trig test were 81 and 7. On which test did she do relatively better? Justify your answer.

Maria's friend Ben had a z-score of 2.25 on the Chemistry test.

a) Interpret this z-score in context.

   Interpretation: Ben's test score was ______________ standard deviations above the mean.

b) What was his actual score on the test?

Example 2

The prices of the printers in a store have a mean of $240 and a standard deviation of $50. The printer that you eventually choose costs $340.

a) What is the z-score for the price of your printer?

b) Interpret this z-score in context.

Practice

Ashish's height is 63 inches. The mean height for boys at his school is 68.1 inches, and the standard deviation is 2.8 inches.

a) What is the z-score for Ashish's height?

b) Interpret the z-score in context.

Using the Standard Normal Curve

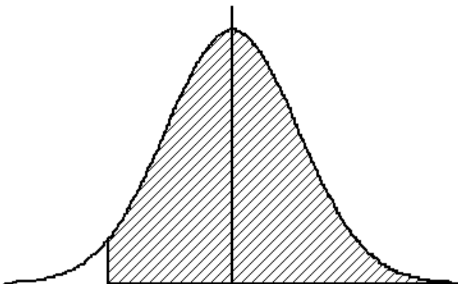
Recall that a normal distribution is a distribution that is symmetric and mound-shaped, as shown at the right.

When calculating probabilities associated with normal distributions, z-scores are used.

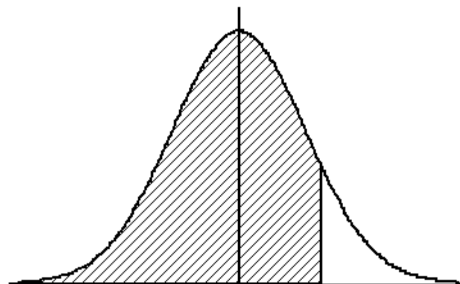
Example 3

The standard normal distribution is the normal distribution with a mean of 0 and a standard deviation of 1. The diagrams below show standard normal distribution curves. Use the table of standard normal probabilities at the end of the notes to determine the shaded areas.

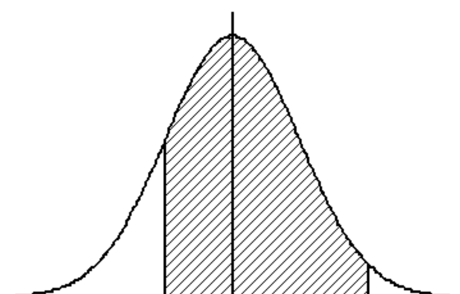
a)



b)



c)



Example 4

For each question, 1) sketch and shade a normal curve to model the situation, and 2 use a table of standard normal probabilities to find the following:

The area to the left of $z = 1.25$

The area to the right of $z = 0.73$

The area to the left of $z = -0.91$

The area to the right of $z = 2.14$

The area between $z = -1.07$ and $z = 3.09$

Example 5

A swimmer named Amy specializes in the 50-meter backstroke. In competition, her mean time for the event is 39.7 seconds, and the standard deviation of her times is 2.3 seconds. Assume that Amy's times are approximately normally distributed.

Use a sketch of a normal curve, z-scores, and the table of standard normal probabilities to answer the questions below.

a) Find the probability that Amy's time in her next race is between 37 and 44 seconds.

b) Find the probability that Amy's time in her next race is more than 45 seconds.

c) What is the probability that Amy's time would be at least 45 seconds?

d) Find the probability that Amy's time in her next race is less than 36 seconds.

Example 6

The distribution of lifetimes of a particular brand of car tires has a mean of 51,200 miles and a standard deviation of 8,200 miles. Assuming that the distribution of lifetimes is approximately normally distributed, find the probability of each event. (Be sure to provide a sketch for each.)

a) A randomly selected tire lasts between 55,000 and 65,000 miles.

b) A randomly selected tire lasts less than 48,000 miles.

c) A randomly selected tire lasts at least 41,000 miles.

d) Explain the meaning of the probability that you found in part (c).

e) What is the probability that the lifetime of a randomly selected tire is within 10,000 miles of the mean lifetime for tires of this brand?

Classwork/Practice

1. The weights of cars passing over a bridge have a mean of $3{,}550$ pounds and standard deviation of 870 pounds. Assume that the weights of the cars passing over the bridge are approximately normally distributed. For a-c, determine the probability of each instance, showing your sketch and your work.

a) The weight of a randomly selected car is more than $4{,}000$ pounds.

b) The weight of a randomly selected car is less than $3{,}000$ pounds.

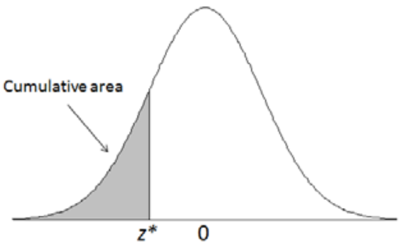
c) The weight of a randomly selected car is between $2{,}800$ and $4{,}500$ pounds.

d) Your friend's car has a weight with a z-score of -1.25. Interpret the z-score in context.

e) For the car in (d), find the actual weight of the car.

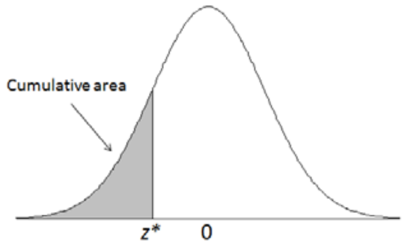
f) Use the **Empirical Rule** to *estimate* the proportion of cars passing over the bridge that weigh between 940 pounds and 5290 pounds. Support your answer with a sketch.

Standard Normal Probabilities



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
–3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
–3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
–3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
–3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
–3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
–3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
–3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
–3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
–2.1	0.0179	0.0174	0.0160	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
–2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
–1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
–1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0599
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
–0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
–0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
–0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
–0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
–0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Standard Normal Probabilities



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999