Lesson 10: Graphing Polynomials

Classwork

Opening Exercise

An engineer is designing a roller coaster for younger children and has tried some functions to model the height of the roller coaster during the first 300 yards. She came up with the following function to describe what she believes would make a fun start to the ride:

$$H(x) = -3x^4 + 21x^3 - 48x^2 + 36x,$$

where $H(x)$ is the height of the roller coaster (in yards) when the roller coaster is $100x$ yards from the beginning of the ride. Graphing the function is a good way to decide how enjoyable this rollercoaster might be for children.

[source for picture: http://www.clipartlord.com/category/structures-clip-art/theme-park-clip-art/]

1. A. Use one of the factoring techniques from Lessons 8 or 9 to factor the equation, given that 3 is one of the zeros of this equation.

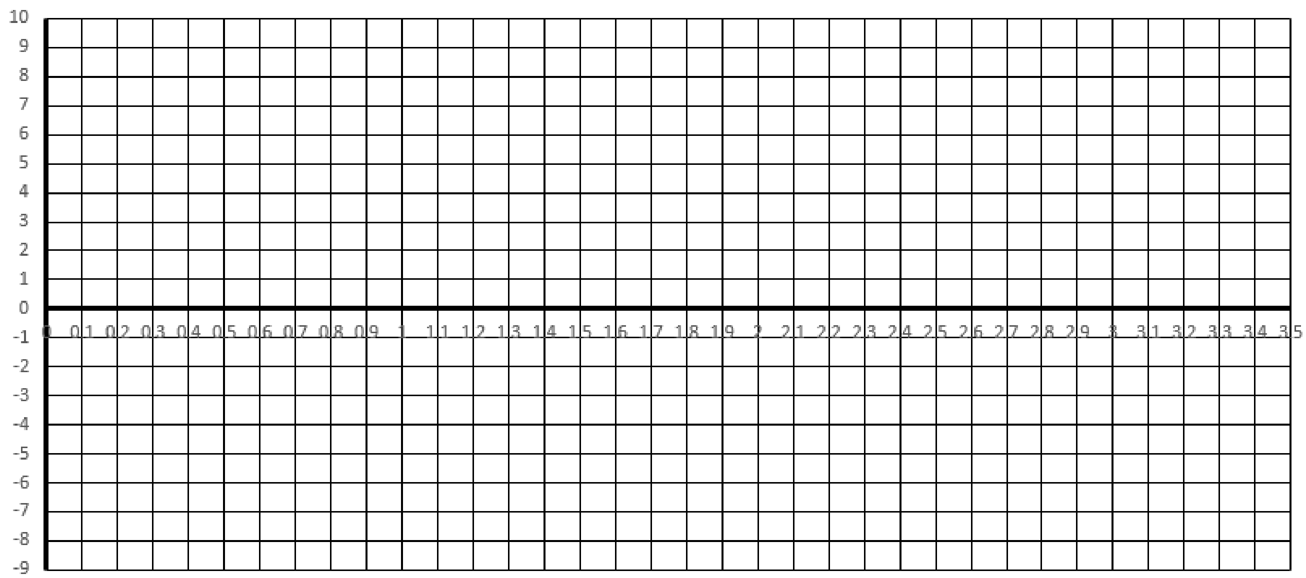
$$H(x) = -3x^4 + 21x^3 - 48x^2 + 36x$$

B. Set $H(x)$ equal to 0 and determine the zeros of the function. Then determine the multiplicities.

C. Determine the end behavior of the function.

2. A. Graph each of the zeros and then use the end behavior and multiplicities to make a rough sketch of the graph. Be sure to use a pencil so that you can erase if necessary in Part (B).

The Children's Rollercoaster



B. To make the graph a little more accurate, choose two points between each zero to substitute into the equation. Write these new points in the table below and then graph them. Erase and reconnect points as needed to make the function smooth and continuous.

x					
$H(x)$					

Discussion

3. Answer the following questions to help determine how enjoyable this rollercoaster would be for children. Remember that the height is in yards and the distance from the beginning of the ride is in 100 yard increments.

   A. Does this function describe a roller coaster that would be fun to ride? Explain.

   B. Can you see any obvious x-values from the equation where the roller coaster is at height 0?

   C. Why do roller coasters always start with the largest hill first?

   D. How do you think the engineer came up with the function for this model?

   E. What is wrong with this roller coaster model at distance 0 yards and 300 yards? Why might this not initially bother the engineer when she is first designing the track?

   F. What would be an appropriate domain for this function? Explain your thinking.

4. Consider the function $f(x) = x^3 - 13x^2 + 44x - 32$.

A. Use the fact that $x - 4$ is a factor of f to factor this polynomial.

B. Find the x-intercepts for the graph of f.

C. At which x-values can the function change from being positive to negative or from negative to positive?

D. To sketch a graph of f, we need to consider whether the function is positive or negative on the four intervals $x < 1$, $1 < x < 4$, $4 < x < 8$, and $x > 8$. Why is that?

E. How can we tell if the function is positive or negative on an interval between x-intercepts?

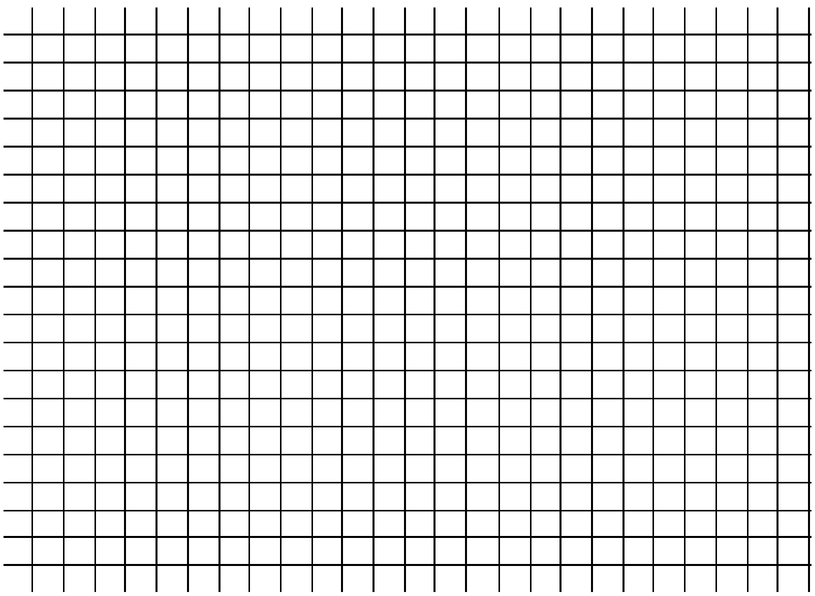
F. For $x < 1$, is the graph above or below the x-axis? How can you tell?

G. For $1 < x < 4$, is the graph above or below the x-axis? How can you tell?

H. For $4 < x < 8$, is the graph above or below the x-axis? How can you tell?

I. For $x > 8$, is the graph above or below the x-axis? How can you tell?

J. Use the information generated in Parts F – I to sketch a graph of f.



K. Graph $y = f(x)$ on the interval from $[0,9]$ using a graphing utility, and compare your sketch with the graph generated by the graphing utility.

Discussion

5. By manipulating a polynomial function into its factored form, we can identify the zeros of the function as well as identify the general shape of the graph. Thinking about the Opening Exercise, what else can we say about the polynomial function and its graph?

   A. The degree of the polynomial function H is 4. How can you find the degree of a function from its factored form?

   B. How many *x*-intercepts does the graph of the polynomial function have?

   C. Note that there are four factors, but only three *x*-intercepts. Why is that?

   D. Can you make one change to the polynomial function such that the new graph would have four *x*-intercepts?

Lesson Summary

Graphing a Polynomial

1. Try to ____________ the polynomial as much as possible to determine the ________.
2. Determine the __________________ of each zero.
3. Use the leading coefficient and the degree of the function to determine the _____ ________________.
4. Find the ____-intercept.
5. Use the equation to find a few more ___________ on the graph.

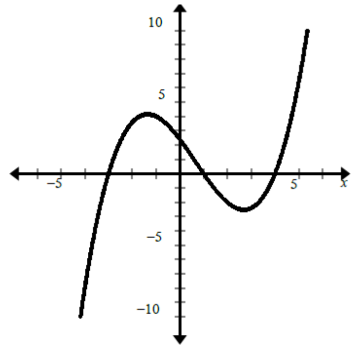
Homework Problem Set

1. Sketch each of the following polynomial functions. What are the function's zeros and multiplicities? What is the end behavior of the function? Choose a point or two to get a more accurate graph.

   A. $f(x) = x(x-1)(x+1)$

   B. $f(x) = (x-1)(x-2)(x+3)(x+4)(x+4)$

2. A. What are the zeros of the polynomial shown?



B. What type of multiplicity must each zero have? Why?

3. The table below lists all the real zeros of a 5th degree polynomial function, $f(x)$, and the multiplicity of each zero. Write two possible equations for $f(x)$.

Zero	Multiplicity
-4	1
-1	2
2	2

Equation 1: ________________________

Equation 2: ________________________

4. Graph the polynomial function $f(x) = x^5 - 8x^3 + 16x$.