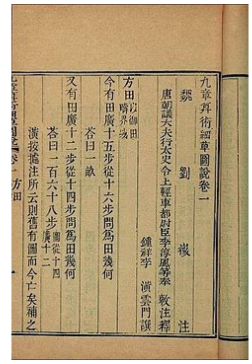
Lesson 11: Graphing Polynomials – Digging Deeper

Opening Exercise

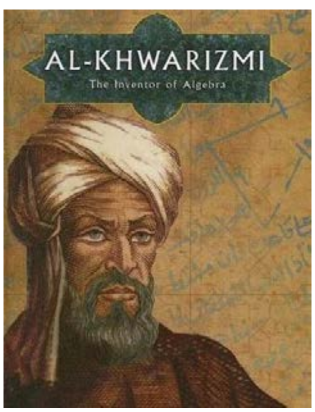
The History of Polynomials



Determining the roots of polynomials, or "solving algebraic equations", is among the oldest problems in mathematics. However, the elegant and practical notation we use today only developed beginning in the 15th century. Before that, equations were written out in words. For example, an algebra problem from the Chinese Arithmetic in Nine Sections, circa 200 BCE, begins "Three sheafs of good crop, two sheafs of mediocre crop, and one sheaf of bad crop are sold for 29 dou." We would write $3x + 2y + z = 29$.

[source: https://en.wikipedia.org/wiki/Polynomial]

The earliest known people to solve polynomial equations were the Ancient Egyptians and Babylonians, although they didn't see what they were solving as equations, that terminology came later. They were able to solve linear ($ax = b$), quadratic ($ax^2 + bx = c$), and indeterminate equations ($x^2 + y^2 = z^2$), and the way we solve them today is still quite similar to their methods.



Later, Alexandrian mathematicians… took the ideas that the Egyptians and Babylonians had come up with and expanded upon them. Their knowledge became a staple of Islamic world, where it became known as "the science of restoration and balancing." The Arabic word for restoration, al-jabru, became the root for the word algebra. In the 9th century, the Arab mathematician al-Kwharizmi wrote one of the first books on Arabic algebra, and it provided examples and proofs of what we now know to be basic algebraic theory. By the end of the 9th century, another Arab mathematician, Abu Kamil, had expanded even further on al-Kwharizmi's theories and was able to prove the basic laws and identities of algebra and solve more complicated problems.

[source: https://hs-mathematics.wikispaces.com/Polynomials]

[picture source: http://www.muslimheritage.com/article/contribution-al-khwarizmi-mathematics-and-geography]

Reflecting on the Reading

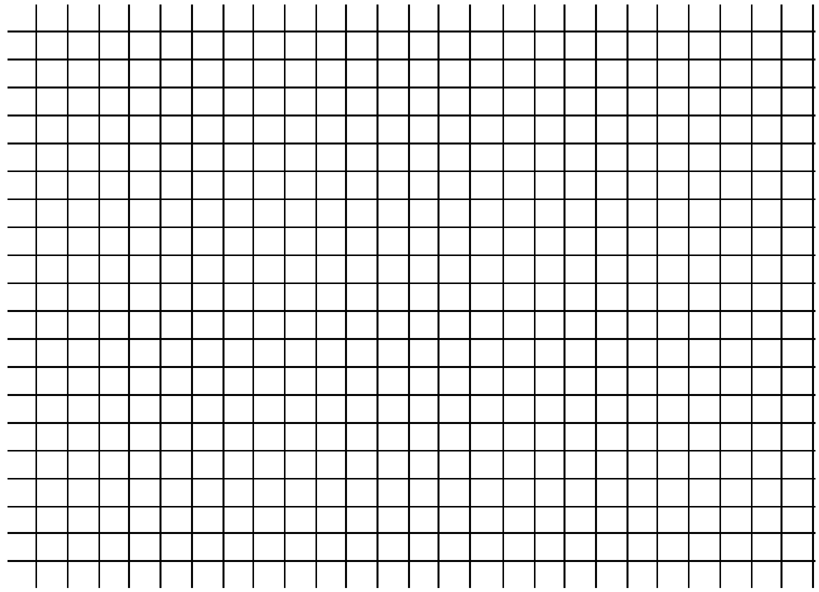
1. Why was the use of symbols instead of words an important breakthrough in mathematics?

2. Name two different applications of polynomials in the real world.

3. Consider the function $f(x) = x^4 - 5x^3 + 7x^2 - 5x + 6$.

A. Use the fact that $x - 3$ is a factor of f to factor this polynomial.

B. Graph the function using the x-intercepts and the end behavior of the graph of f.



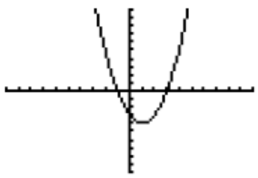
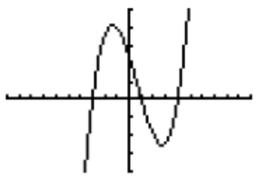
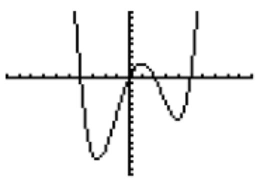
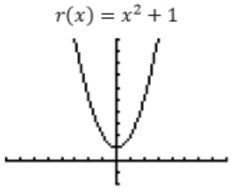
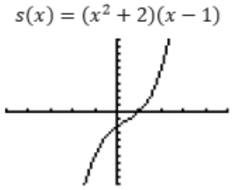
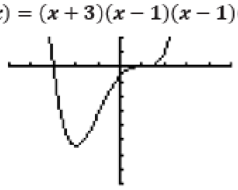
C. How can we tell if the function is positive or negative on an interval between x-intercepts **without** using the graph?

Without using the graph, answer Parts D – F.

	Interval	Is the graph above or below the x-axis?	How can you tell?
D.	$x < 2$		
E.	$2 < x < 3$		
F.	$x > 3$		

H. Did the answers you wrote in Parts D – F agree with your graph?

4. Consider the following polynomial functions in factored form and their graphs. Fill in the table.

	Graph and Equation	Degree of the Polynomial	Number of x-intercepts	Number of Relative Maximum and Relative Minimum Points on the Graph
A.	$f(x) = (x + 1)(x - 3)$ 			
B.	$g(x) = (x + 3)(x - 1)(x - 4)$ 			
C.	$h(x) = (x)(x + 4)(x - 2)(x - 5)$ 			
D.	$r(x) = x^2 + 1$ 			
E.	$s(x) = (x^2 + 2)(x - 1)$ 			
F.	$\boldsymbol{t(x) = (x + 3)(x - 1)(x - 1)(x - 1)}$ 			

Discussion

5. For any particular polynomial, can we determine how many relative maxima or minima there are? What observations can we make from this information? Is this true for every polynomial?

Lesson Summary

A polynomial of degree n may have up to _____ x-intercepts and up to _______ relative maximum/minimum points.

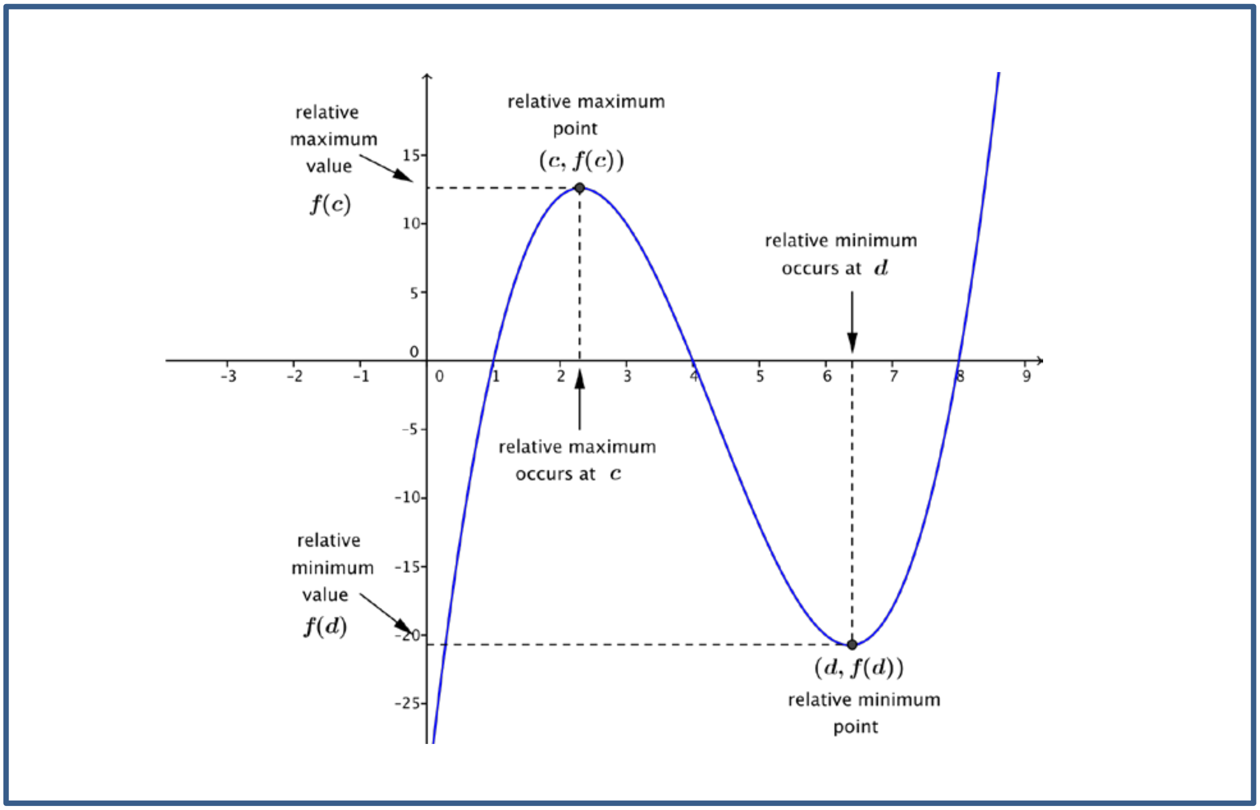
INCREASING/DECREASING: The function, f, is . . .

***increasing** on the interval* I if $f(x_1)$______$f(x_2)$ whenever $x_1 < x_2$ in I.	***decreasing** on the interval* I if $f(x_1)$______$f(x_2)$ whenever $x_1 < x_2$ in I.

RELATIVE MAXIMUM AND MINIMUM: The function, f, has a . . .

*relative **maximum** at* c if there exists an open interval I of the domain that contains c such that $f(x)$____$f(c)$ for all x in the interval I. If f has a relative maximum at c, then the value $f(c)$ is called the *relative maximum value*.	*relative **minimum** at* c if there exists an open interval I of the domain that contains c such that $f(x)$____$f(c)$ for all x in the interval I. If f has a relative minimum at c, then the value $f(c)$ is called the *relative minimum value*.

The plural of maximum is **maxima**, and the plural of minimum is **minima**.



Homework Problem Set

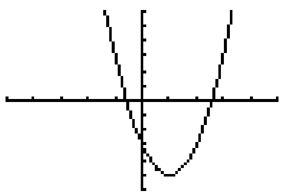
1. For each function below, identify the largest possible number of x-intercepts and the largest possible number of relative maxima and minima based on the degree of the polynomial. Then use a calculator or graphing utility to graph the function and find the actual number of x-intercepts and relative maxima and minima.

$f(x) = 4x^3 - 2x + 1$ $\quad$ $g(x) = x^7 - 4x^5 - x^3 + 4x$ $\quad$ $h(x) = x^4 + 4x^3 + 2x^2 - 4x + 2$

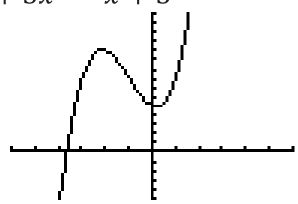
Function	Largest number of x-intercepts	Largest number of relative max/min	Actual number of x-intercepts	Actual number of relative max/min
A. f				
B. g				
C. h				

2. We have talked about x-intercepts of the graph of a function in both this lesson and the previous one. The x-intercepts correspond to the zeros of the function. Consider the following examples of polynomial functions and their graphs to determine an easy way to find the y-intercept of the graph of a polynomial function.

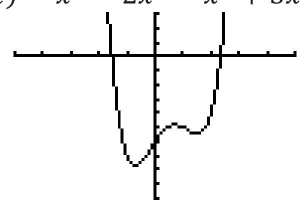
$f(x) = 2x^2 - 4x - 3$



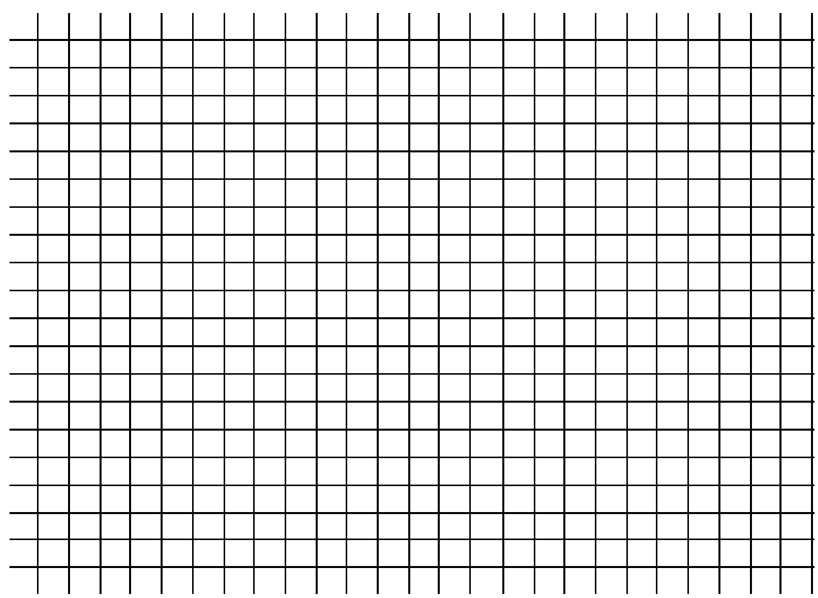
$f(x) = x^3 + 3x^2 - x + 5$



$f(x) = x^4 - 2x^3 - x^2 + 3x - 6$



3. Sketch a graph of the function $f(x) = \frac{1}{2}(x+5)(x+1)(x-2)$ by finding the zeros and determining the sign of the values of the function between zeros.



4. Sketch a graph of the function
$f(x) = -(x+2)(x-4)(x-6)$
by finding the zeros and determining the sign of the values of the function between zeros.

5. Sketch a graph of the function
$f(x) = (x+3)(x+3)(x+3)(x+3)$
by finding the zeros and determining the sign of the values of the function between zeros.

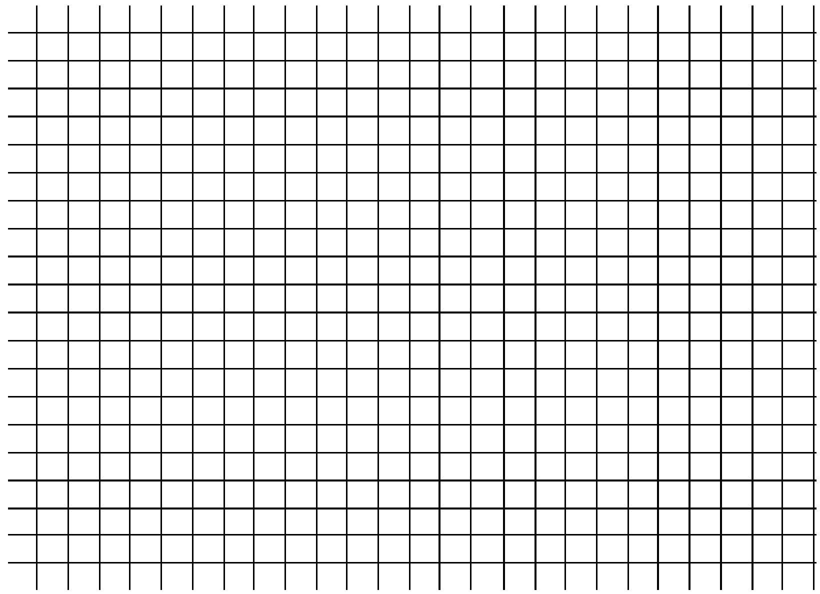
6. Sketch a graph of the function
   $f(x) = x^3 - 2x^2 - x + 2$
   by finding the zeros and determining the sign of the values of the function between zeros.

7. Sketch a graph of the function
   $f(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$
   by determining the sign of the values of the function between the zeros -1, 1, and 3.

8. A function f has zeros at -1, 3, and 5.

   We know that $f(-2)$ and $f(2)$ are negative, while $f(4)$ and $f(6)$ are positive.

   Sketch a graph of f.



9. The function $h(t) = -16t^2 + 33t + 45$ represents the height of a ball tossed upward from the roof of a building 45 feet in the air after t seconds. Without graphing, determine when the ball will hit the ground.