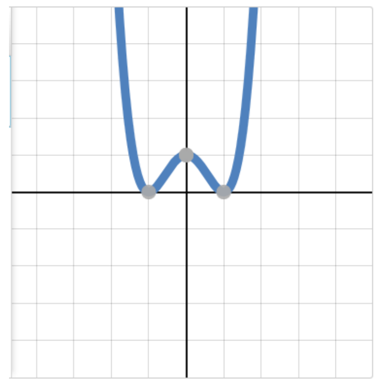
Exit Ticket Sample Solutions

There is a 4th–degree polynomial function with zeros at -1 and 1 and a relative maximum at (0, 1). Sketch a possible graph and write a possible equation for this function.

One possible solution is $f(x) = (x + 1)^2(x - 1)^2$.



Homework Problem Set Solutions

A graphing calculator or access to the internet for Desmos is needed to verify student answers for Homework Problem Set #1. You may need to do this verification in class.

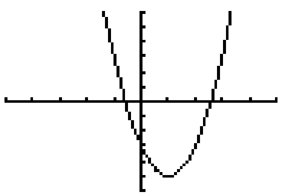
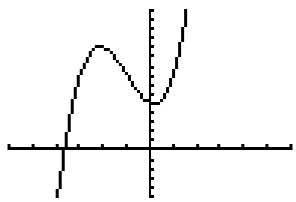
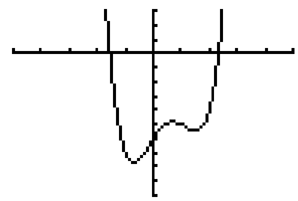
1. **For each function below, identify the largest possible number of x-intercepts and the largest possible number of relative maxima and minima based on the degree of the polynomial. Then use a calculator or graphing utility to graph the function and find the actual number of x-intercepts and relative maxima and minima.**

$f(x) = 4x^3 - 2x + 1$ $\quad$ $g(x) = x^7 - 4x^5 - x^3 + 4x$ $\quad$ $h(x) = x^4 + 4x^3 + 2x^2 - 4x + 2$

Function	Largest number of x-intercepts	Largest number of relative max/min	Actual number of x-intercepts	Actual number of relative max/min
A. f	3	2	1	2
B. g	7	6	5	4
C. h	4	3	0	3

2. **We have talked about x-intercepts of the graph of a function in both this lesson and the previous one. The x-intercepts correspond to the zeros of the function. Consider the following examples of polynomial functions and their graphs to determine an easy way to find the y-intercept of the graph of a polynomial function.**

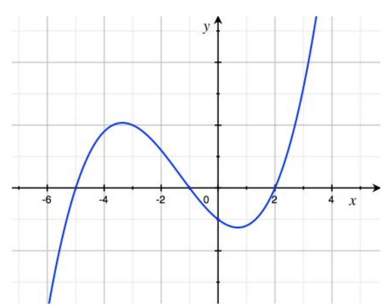
$f(x) = 2x^2 - 4x - 3$ $\quad$ $f(x) = x^3 + 3x^2 - x + 5$ $\quad$ $f(x) = x^4 - 2x^3 - x^2 + 3x - 6$

The y-intercept is the value where the graph of a function f intersects the y-axis, if 0 is in the domain of f. Therefore, for a function f whose domain and range are a subset of the real numbers, the y-intercept is $f(0)$.

For polynomial functions, $f(0)$ is easy to determine—it is just the constant term when the polynomial function is written in standard form.

3. **Sketch a graph of the function $f(x) = \frac{1}{2}(x+5)(x+1)(x-2)$ by finding the zeros and determining the sign of the values of the function between zeros.**



The zeros are -5, -1, and 2.

For $x < -5$: $f(-6) = -20$, so the graph is below the x-axis for $x < -5$.

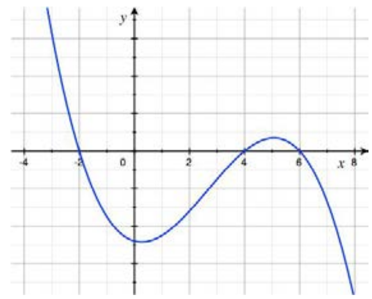
For $-5 < x < -1$: $f(-3) = 10$, so the graph is above the x-axis for $-5 < x < -1$.

For $-1 < x < 2$: $f(0) = -5$, so the graph is below the x-axis for $-1 < x < 2$.

For $x > 2$: $f(3) = 16$, so the graph is above the x-axis for $x > 2$.

4. **Sketch a graph of the function $f(x) = -(x+2)(x-4)(x-6)$ by finding the zeros and determining the sign of the values of the function between zeros.**

The zeros are -2, 4, and 6.



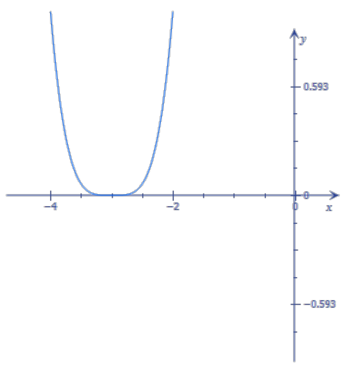
For $x < -2$: $f(-3) = 63$, so the graph is above the x-axis for $x < -2$.

For $-2 < x < 4$: $f(0) = -48$, so the graph is below the x-axis for $-2 < x < 4$.

For $4 < x < 6$: $f(5) = 7$, so the graph is above the x-axis for $4 < x < 6$.

For $x > 6$: $f(7) = -27$, so the graph is below the x-axis for $x > 6$.

5. **Sketch a graph of the function $f(x) = (x+3)(x+3)(x+3)(x+3)$ by finding the zeros and determining the sign of the values of the function between zeros.**



Zeros: $-3, -3, -3, -3$ (repeated zero)

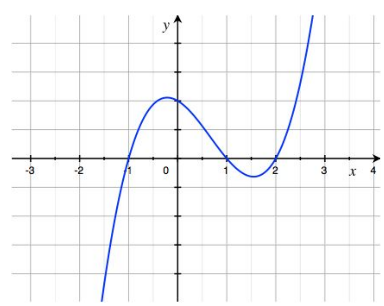
Solutions to $f(x) = 0$: -3

x-intercept: -3

The degree is 4, which is greater than the number of x-intercepts.

6. **Sketch a graph of the function $f(x) = x^3 - 2x^2 - x + 2$ by finding the zeros and determining the sign of the values of the function between zeros.**

We can factor by grouping to find $f(x) = (x^2 - 1)(x-2)$. The zeros are -1, 1, and 2.



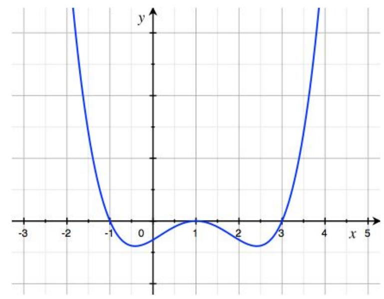
For $x < -1$: $f(-2) = -12$, so the graph is below the x-axis for $x < -1$.

For $-1 < x < 1$: $f(0) = 2$, so the graph is above the x-axis for $-1 < x < 1$.

For $1 < x < 2$: $f\left(\frac{3}{2}\right) = -\frac{5}{8}$, so the graph is below the x-axis for $1 < x < 2$.

For $x > 2$: $f(3) = 8$, so the graph is above the x-axis for $x > 2$.

7. Sketch a graph of the function $f(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$ by determining the sign of the values of the function between the zeros -1, 1, and 3.



We are told that the zeros are -1, 1, and 3.

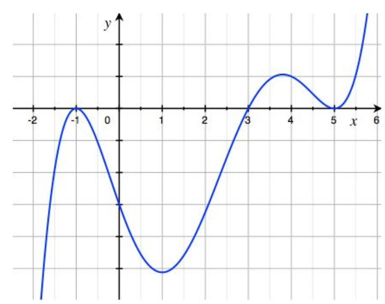
For $x < -1$: $f(-2) = 45$, so the graph is above the x-axis for $x < -1$.

For $-1 < x < 1$: $f(0) = -3$, so the graph is below the x-axis for $-1 < x < 1$.

For $1 < x < 3$: $f(2) = -3$, so the graph is below the x-axis for $1 < x < 3$.

For $x > 3$: $f(4) = 45$, so the graph is above the x-axis for $x > 3$.

8. A function f has zeros at -1, 3, and 5. We know that $f(-2)$ and $f(2)$ are negative, while $f(4)$ and $f(6)$ are positive. Sketch a graph of f.



From the information given, the graph of f lies below the x-axis for $x < -1$ and $-1 < x < 3$ and that it touches the x-axis at -1. Similarly, we know that the graph of f lies above the x-axis for $3 < x < 5$ and $5 < x$ and that it touches the x-axis at 5. We also know that the graph crosses the x-axis at 3.

9. The function $h(t) = -16t^2 + 33t + 45$ represents the height of a ball tossed upward from the roof of a building 45 feet in the air after t seconds. Without graphing, determine when the ball will hit the ground.

Factor: $h(t) = (t - 3)(-16t - 15)$

Solve $h(t) = 0$: $(t - 3)(-16t - 15) = 0$

$t = 3$ seconds or $t = -\frac{15}{16}$ seconds.

The ball hits the ground at time 3 seconds; the solution $-\frac{15}{16}$ does not make sense in the context of the problem.