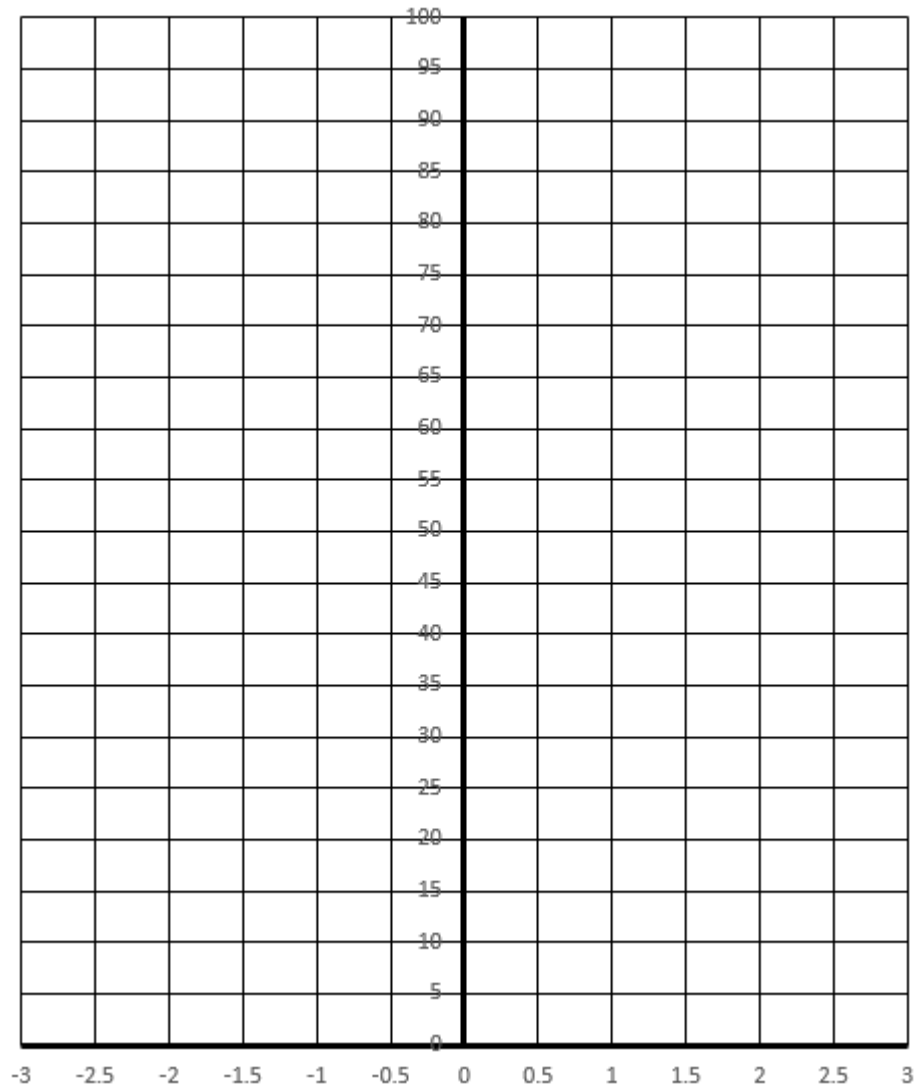


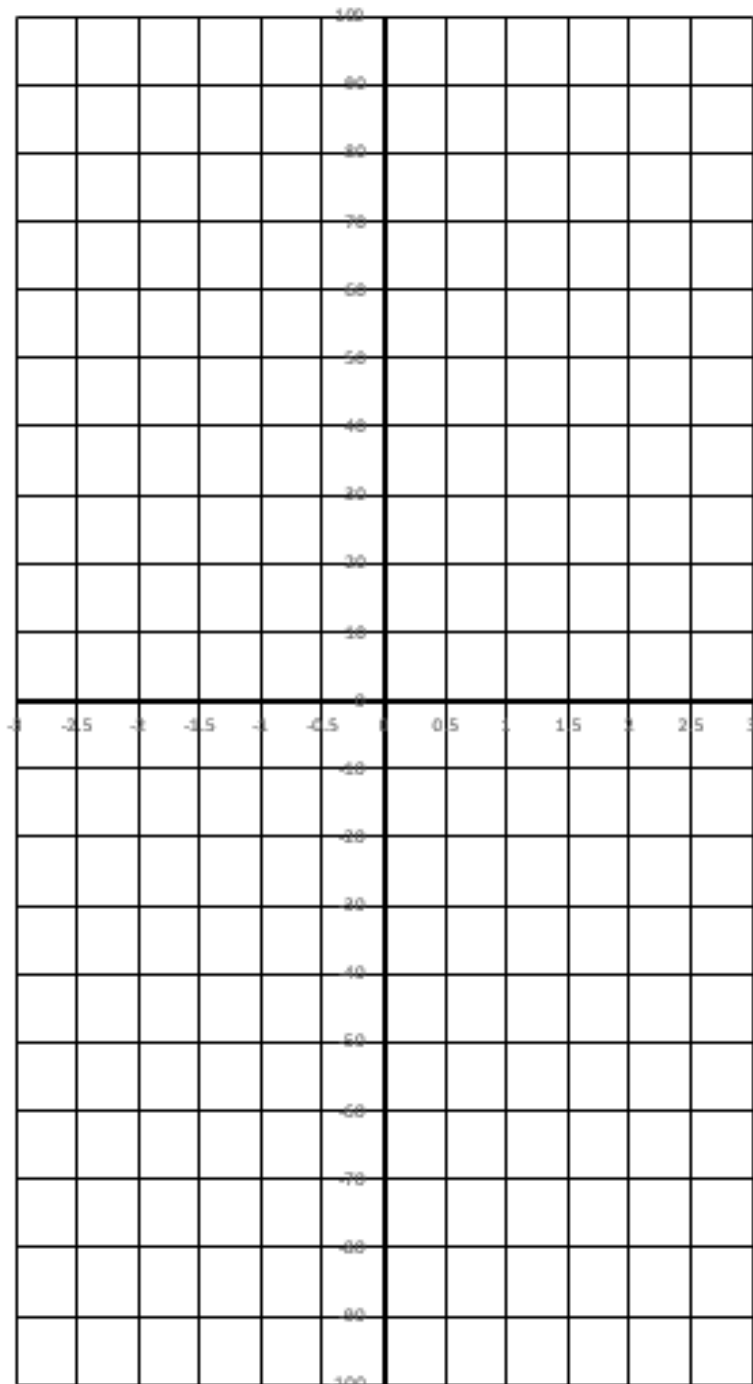
Lesson 12: Structure in Graphs of Polynomial Functions

Opening Exercise

- A. Sketch the graph of $f(x) = x^2$.
- B. What will the graph of $g(x) = x^4$ look like? Sketch it on the same coordinate plane.
- C. What will the graph of $h(x) = x^6$ look like? Sketch it on the same coordinate plane.



2. A. Sketch the graph of $f(x) = x^3$.
- B. What will the graph of $g(x) = x^5$ look like? Sketch this on the same coordinate plane.
- C. What will the graph of $h(x) = x^7$ look like? Sketch this on the same coordinate plane.



3. Based on the graphs from Exercises 1 and 2, what observations can you make about functions that have the form $f(x) = x^{\text{even number}}$ and $f(x) = x^{\text{odd number}}$?

4. Consider the following function, $f(x) = 2x^4 + x^3 - x^2 + 5x + 3$, with a mixture of odd and even degree terms.
- A. Predict whether its end behavior will be like the function in Exercise 1 or Exercise 2.
- B. Graph the function f using a graphing utility to check your prediction.
5. Consider the following function, $f(x) = 2x^5 - x^4 - 2x^3 + 4x^2 + x + 3$, with a mixture of odd and even degree terms.
- A. Predict whether its end behavior will be like the function in Exercise 1 or Exercise 2.
- B. Graph the function f using a graphing utility to check your prediction.
6. A. Thinking back to our discussion of x -intercepts of graphs of polynomial functions from the previous lesson, sketch a graph of an even-degree polynomial function that has no x -intercepts.
- B. Similarly, can you sketch a graph of an odd-degree polynomial function with no x -intercepts?

A function f is an *even function* if it satisfies the equation $f(-x) = f(x)$ for every number x in its domain.

7. Consider the function $f(x) = -3x^2 + 7$. Note that the degree of the function is even, and each term is of an even degree (the constant term is degree 0).

A. Graph the function using a graphing utility.

B. Describe the symmetry this graph displays.

C. Evaluate $f(-x)$.

$$f(x) = -3x^2 + 7$$

$$f(-x) = -3(\underline{\hspace{1cm}})^2 + 7$$

$$= \underline{\hspace{2cm}}$$

D. Is f an even function? Explain how you know.

8. Determine if the quadratic function $f(x) = 2x^2 - x + 3$ is an even function. (Use Exercise 7C as a model for the work that must be shown.)

A function f is an *odd function* if it satisfies the equation $f(-x) = -f(x)$ for every number x in its domain.

9. Consider the function $f(x) = 3x^3 - 4x$. The degree of the function is odd, and each term is of an odd degree.

- A. Graph the function using a graphing utility.
- B. Describe the symmetry this graph displays.

C. Evaluate $f(-x)$.

$$f(x) = 3x^3 - 4x$$

$$f(-x) = 3(\underline{\hspace{1cm}})^3 - 4(\underline{\hspace{1cm}})$$

$$= \underline{\hspace{2cm}}$$

D. Is f an odd function? Explain how you know.

Lesson Summary

EVEN FUNCTION: The function f is called *even* if the equation $f(x) = f(-x)$ is true for every number x in the domain.

Even-degree polynomial functions are sometimes even functions, like $f(x) = x^{10}$, and sometimes not, like $g(x) = x^2 - x$. The leading term **must** have an even exponent.

ODD FUNCTION: The function f is called *odd* if the equation $f(-x) = -f(x)$ is true for every number x in the domain.

Odd-degree polynomial functions are sometimes odd functions, like $f(x) = x^{11}$, and sometimes not, like $h(x) = x^3 - x^2$. The leading term **must** have an odd exponent.

Homework Problem Set

1. The National Agricultural Statistics Service (NASS) is an agency within the USDA that collects and analyzes data covering virtually every aspect of agriculture in the United States. The following table contains information on the amount (in tons) of the following vegetables produced in the U.S. from 1988–1994 for processing into canned, frozen, and packaged foods: lima beans, snap beans, beets, cabbage, sweet corn, cucumbers, green peas, spinach, and tomatoes.

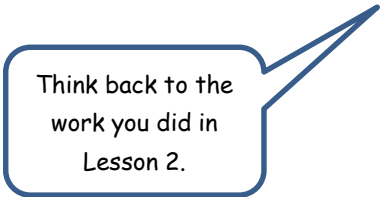
Vegetable Production by Year

Year	Vegetable Production (tons)
1988	11,393,320
1989	14,450,860
1990	15,444,970
1991	16,151,030
1992	14,236,320
1993	14,904,750
1994	18,313,150

Source: NASS Statistics of Vegetables and Melons, 1995, Table 191.

http://www.nass.usda.gov/Publications/Ag_Statistics/1995-1996/agr95_4.pdf

- A. Plot the data using a graphing utility. Then sketch the graph in the space below.



Think back to the work you did in Lesson 2.

- B. Determine if the data display the characteristics of an odd- or even-degree polynomial function.
- C. List two possible reasons the data might have such a shape.

2. The U.S. Energy Information Administration (EIA) is responsible for collecting and analyzing information about energy production and use in the United States and for informing policy makers and the public about issues of energy, the economy, and the environment. The following table contains data from the EIA about natural gas consumption from 1950–2010, measured in millions of cubic feet.

U.S. Natural Gas Consumption by Year

Year	U.S. natural gas total consumption (millions of cubic feet)
1950	5.77
1955	8.69
1960	11.97
1965	15.28
1970	21.14
1975	19.54
1980	19.88
1985	17.28
1990	19.17
1995	22.21
2000	23.33
2005	22.01
2010	24.09

Source: U.S. Energy Information Administration. <http://www.eia.gov/dnav/ng/hist/n9140us2a.htm>

- A. Plot the data using a graphing utility. Then sketch the graph in the space below.
- B. Determine if the data display the characteristics of an odd- or even-degree polynomial function.
- C. List two possible reasons the data might have such a shape.

3. The Center for Transportation Analysis (CTA) studies all aspects of transportation in the United States, from energy and environmental concerns to safety and security challenges. A 1997 study compiled the following data of the fuel economy in miles per gallon (mpg) of a car or light truck at various speeds measured in miles per hour (mph). The data are compiled in the table below.

Fuel Economy by Speed

Speed (mph)	Fuel Economy (mpg)
15	24.4
20	27.9
25	30.5
30	31.7
35	31.2
40	31.0
45	31.6
50	32.4
55	32.4
60	31.4
65	29.2
70	26.8
75	24.8

Source: Transportation Energy Data Book, Table 4.28. <http://cta.ornl.gov/data/chapter4.shtml>

- a. Plot the data using a graphing utility. Then sketch the graph in the space below. Which variable is the independent variable?
- b. This data can be modeled by a polynomial function. Determine if the function that models the data would have an even or odd degree.
- c. Is the leading coefficient of the polynomial that can be used to model this data positive or negative?
- d. List two possible reasons the data might have the shape that it does.

4. For each problem below, determine if the function is even, odd or neither. Be sure to show your work.

A. $f(x) = 4x^3 - 2x$	B. $g(x) = 5x^3 - 7x^2 + 2$
C. $h(x) = -x^4 + 6x^2 + 9$	D. $j(x) = 3x^2 + x$

5. In Exercises 7 and 9 (in the lesson), you looked at the symmetry of the graphs. Even functions are symmetry over the y -axis, while odd functions are symmetry about the origin (rotational symmetry). Use this idea to determine which of the following functions are even, odd or neither.

