Lesson 13: The Juice Can Problem

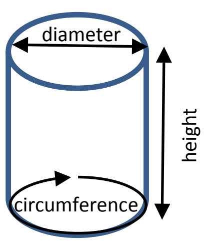
In the last unit, we looked at how we could maximize the volume of an open-topped box. In this unit we'll explore how the surface area of a cylinder can be minimized.

Discussion

1. Why would a manufacturer of juice want the least surface area for their juice can?

Exploration Activity

The Hart Vita-Drink Company wants to produce a "fun-size" can of their most popular drink, "Hart Wholesome Veggie Juice". There are several radius sizes they are considering and the heights will vary, but all of them must have a volume of 200 cm^3. There are several important formulas from geometry you'll need to help you determine the dimensions of the juice can that has the minimum surface area.



Volume of a Cylinder	Surface Area of a Cylinder	Circumference of a Circle
$V = \pi r^2 h$	$SA = 2\pi r^2 + 2\pi rh$	$C = 2\pi r$

2. A. If the radius is 5 cm, what would be the height of the cylinder with a volume of 200 cm^3? Round your answer to the nearest tenth.

   B. If the radius is 5 cm, what would be the surface area of the cylinder with a volume of 200 cm^3? Round your answer to the nearest tenth.

   C. Why might these **not** be good dimensions for a juice can?

Your group will need: Cylinder Base Handout, one pair of scissors, a ruler, 1 sheet of cardstock or construction paper and tape

3. As we saw in the last unit, it helps to see a model of the object you are analyzing.

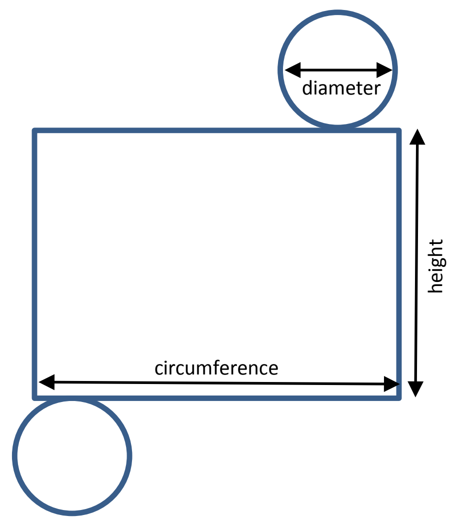
   A. Use the handout to create a cylinder with the given circular base. Your group will need to determine the height and circumference of the lateral face of your cylinder. Round all answers to the nearest tenth.

   Our group's radius (of the base): _______________

   The height of our cylinder: ___________________
   (Remember, you must have a volume of 200 cm^3.)

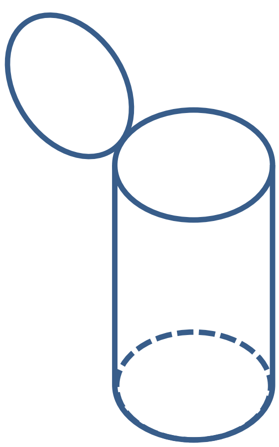
   The circumference of our cylinder: _____________

   B. Write in the dimensions on the net of a cylinder shown at the right.



   C. Use the cardstock or construction paper and a ruler to measure the lateral face of your cylinder. Cut out and tape together the lateral face of your cylinder.

   D. Tape one of the circular bases onto the cylinder, but leave the other only partially taped so that we can fill the cylinder in Exercise 5.



4. Look at all the cylinders created by your class. Which do you believe has the smallest surface area? (Which do you believe used the least amount of cardstock or construction paper?)

5. Do all of the cylinders have the same volume? How can you be sure?

Analyzing the Data

It can be difficult to determine the dimensions of the cylinder with the least surface area just by looking at some examples. We'll gather and graph the data to help us do a better job of determining the minimal surface area.

6. A. Determine the surface area of your group's cylinder. Round your answer to the nearest tenth.

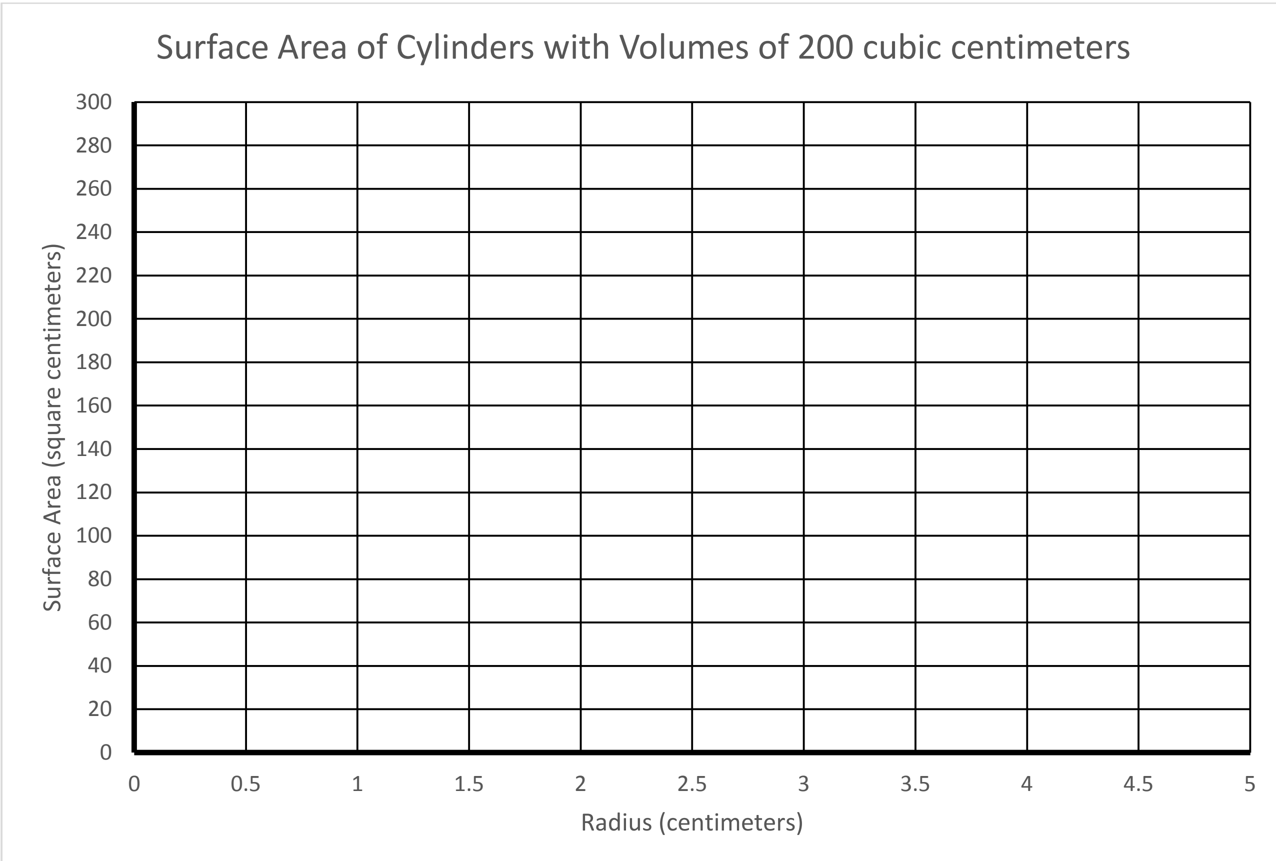
Surface area of our cylinder: ______________

B. Gather data from the other groups and record them and your own data in the table below.

Surface Area Data from Cylinders with a Volume of 200 cm^3

Radius of Cylinder (centimeters)	Height of Cylinder (centimeters)	Surface Area of Cylinder (centimeters^2)
1.5		
1.75		
2		
2.5		
3		
3.5		
4		
4.5		

7. Use the data in the table to create a graph of the radius versus surface area on the grid below.



8. A. Does it make sense to connect the points? Explain your thinking.

   B. Estimate the surface area of a cylinder with a radius of 2.75 centimeters.

   C. Estimate the radius of a cylinder with a surface area of 210 square centimeters. Is there more than one possible cylinder? Explain.

9. Based on your graph, what radius size should be used to create a can with the least surface area? Explain your reasoning. Does this agree with your guess in Exercise 4?

Homework Problem Set

REVIEW

Write a polynomial function that meets the stated conditions.

1. The zeros are -2 and 1.

2. The zeros are -1, 2, and 7.

3. The zeros are $-\frac{1}{2}$ and $\frac{3}{4}$.

4. The zeros are $-\frac{2}{3}$ and 5, and the constant term of the polynomial is -10.

5. The zeros are 2 and $-\frac{3}{2}$, the polynomial has degree 3, and there are no other zeros.

6. Factor each of the following or state that the expression cannot be factored.

A. $x^2 - 5x + 6$

B. $x^2 - 36$

C. $x^2 - 7x + 12$

D. $a^2b^2 - c^2$

E. $a^3 - 27$

F. $a^2 + b^2$

7. Simplify each expression.

A. $\dfrac{a^2+2a-15}{a-3}$

B. $\dfrac{8a^3-27}{2a-3}$

C. $\dfrac{2a^2-2b^2}{a+b}$

D. $\dfrac{3x^3+8x^2+13x+6}{3x+2}$

E. $\dfrac{3x^3-10x^2+13x-10}{x-2}$

F. $\dfrac{8x^3-10x^2+3x}{2x-1}$