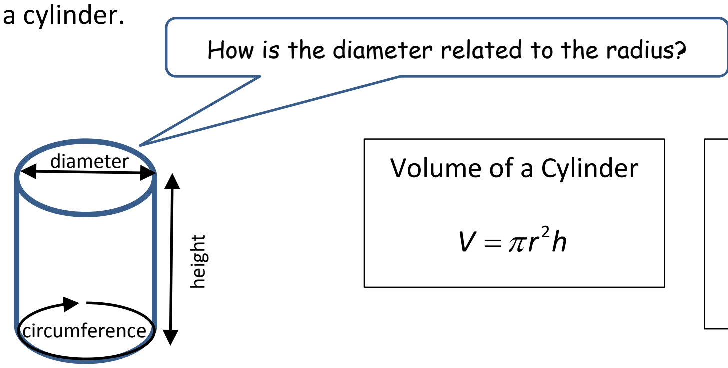
Lesson 14: Rationally Speaking

In Lesson 13, we explored how the surface area of a cylinder could be minimized by using the data in a graph. But what would the equation of this function look like?

Exploration Activity

1. Below are some of the equations we used in Lesson 13 to find the height, circumference and surface area of a cylinder.



Volume of a Cylinder	Surface Area of a Cylinder	Circumference of a Circle
$V=\pi r^2 h$	$SA=2\pi r^2+2\pi rh$	$C=2\pi r$

A. If the volume is 200 cm^3, what is the height in terms of the radius?

B. Use the height expression from Part A to write the surface area formula in terms of the radius. Simplify your new formula.

C. Use the equation from Part B to find the surface area of a cylinder with a volume of 200 cm^3 and a radius of 5 cm. Does this agree with your answer from Lesson 13 Exercise 2? Explain any differences.

D. Is the equation in Part B a quadratic equation? A polynomial equation? How do you know?

Rational Functions, like $S(r)=2\pi r^2+\frac{400}{r}$, are defined as functions that can be written in the form, $f(x)=\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomial functions and $Q(x)\neq 0$. We'll come back to these functions in Lesson 19, but first we need to remember a few things about rational expressions and their operations.

2. With a partner, write two fractions that are equivalent to $\frac{1}{3}$, and then use the rectangles below to create visual models to justify your response. $\frac{1}{3}$ = ☐ = ☐

Discussion

3. A. Circle the following that are rational numbers.

$\frac{3}{4}$	3.14	π	$\frac{5}{0}$	$-\sqrt{17}$
23	$\frac{1+\sqrt{5}}{2}$	-1	6.022×10^{23}	0

B. Of the numbers that were not rational, were they all irrational numbers?

4. Today we'll learn about rational expressions, which are related to the polynomials we've been studying. Just as the integers are the foundational building blocks of rational numbers, polynomial expressions are the foundational building blocks for rational expressions. Based on what we know about rational numbers, give an example of a rational expression.

5. The following are examples of rational expressions. We need to exclude values of the variables that make the denominators zero so that we don't divide by zero. Fill in the rest of the table with the values that must be excluded.

	Rational Expression	Excluded Values
Example	$\frac{31}{47}$	The denominator is never zero, so we do not need to exclude any values.
A.	$\frac{ab^2}{3a-2b}$	
B.	$\frac{5x+1}{3x^2+4}$	
C.	$\frac{3}{b^2-7}$	
D.	$\frac{5-x}{5+x}$	

6. Rodrigo created a Frayer model of rational expressions, but forgot to put in the titles of each box. Write in an appropriate title for each box.

____________________ *An expression that can be written as $\frac{P}{Q}$, where P and Q are polynomials, and Q is not zero.*	____________________ *Follows similar rules as rational numbers do.*
Rational Expression	
$\frac{3}{5}$, $\frac{x^2-4x}{x+1}$ *with* $x \neq -1$ $\frac{x^2y}{2}$, $\frac{a^2+b^2}{(a+2)(a-1)}$, *with* $a \neq -2, 1$ ____________________	$\frac{x+1}{0}$ *(cannot divide by* 0*)* $\frac{2^x}{3x}$ *(*2^x *is not a polynomial)* ____________________

7. It is important to note that the excluded values of the variables remain even after simplification. This is because the two expressions would not be equal if the variables were allowed to take on these values. Discuss with a partner when the following are not equivalent and why, and give one example that would make the two expressions equivalent.

	Equivalent Expressions	**As long as . . .** **(excluded values)**	**One example**
Example	$\frac{2x}{3x}$ and $\frac{2}{3}$	$x \neq 0$	For x = 5, $\frac{2x}{3x} \rightarrow \frac{2 \bullet 5}{3 \bullet 5} = \frac{10}{15} = \frac{2}{3}$
A.	$\frac{3x(x-5)}{4(x-5)}$ and $\frac{3x}{4}$		
B.	$\frac{x-3}{x^2-x-6}$ and $\frac{1}{x+2}$		
C.	$\frac{x^2+4x+4}{x+2}$ and $x+2$		

D. Summarize any conclusions you can draw about equivalent rational expressions.

8. A. Consider the following rational expression: $\frac{2(a-1)-2}{6(a-1)-3a}$. Discuss the following in your group: For what values of a is the expression undefined?

B. Simplify the rational expression : $\frac{2(a-1)-2}{6(a-1)-3a}$ as much as possible.

9. What is the excluded value for the function you wrote in Exercise 1B? Is this a concern in this situation? Explain.

Lesson Summary

If a, b, and n are integers with $n \neq 0$ and $b \neq 0$, then

$$\frac{na}{nb} = \frac{a}{b}$$

The rule for rational expressions is the same as the rule for integers but requires the domain of the rational expression to be restricted (i.e., no value of the variable that can make the denominator of the original rational expression zero is allowed).

Homework Problem Set

1. Reduce the following rational expressions to lowest terms, and identify the values of the variable(s) that must be excluded to prevent division by zero.

   A. $\frac{2(x+1)+2}{(2x+3)(x+1)-1}$

   B. $\frac{x^2-x-6}{5x^2+10x}$

   C. $\frac{3-x}{x^2-9}$

   D. $\frac{3x-3y}{y^2-2xy+x^2}$

2. Find an equivalent rational expression in lowest terms, and identify the value(s) of the variable that must be excluded to prevent division by zero.

a. $\frac{16n}{20n}$	b. $\frac{x^3y}{y^4x}$	c. $\frac{2n-8n^2}{4n}$
d. $\frac{db+dc}{db}$	e. $\frac{x^2-9b^2}{x^2-2xb-3b^2}$	f. $\frac{3n^2-5n-2}{2n-4}$
g. $\frac{3x-2y}{9x^2-4y^2}$	h. $\frac{4a^2-12ab}{a^2-6ab+9b^2}$	i. $\frac{y-x}{x-y}$

j. $\frac{a^2-b^2}{b+a}$	k. $\frac{4x-2y}{3y-6x}$	l. $\frac{9-x^2}{(x-3)^3}$
m. $\frac{x^2-5x+6}{8-2x-x^2}$	n. $\frac{a-b}{xa-xb-a+b}$	o. $\frac{8x^3-y^3}{4x^2-y^2}$

3. Write a rational expression with denominator $6b$ that is equivalent to

A. $\frac{a}{b}$

B. one-half of $\frac{a}{b}$

C. $\frac{1}{3}$

4. Remember that algebra is just a symbolic method for performing arithmetic.

   A. Simplify the following rational expression: $\frac{(x^2y)^2(xy)^3z^2}{(xy^2)^2yz}$.

   B. Simplify the following rational expression without using a calculator: $\frac{12^2 \cdot 6^3 \cdot 5^2}{18^2 \cdot 15}$.

   C. How are the calculations in Parts A and B similar? How are they different? Which expression was easier to simplify?