Lesson 15: Multiplying and Dividing Rational Expressions

Over the next few lessons we'll work with rational expressions and learn to multiply, divide, add and subtract them. We need these skills in order to solve rational equations like the one you wrote in Lesson 14 or the problem below.

Exploratory Exercise

1. Consider an ecosystem of rabbits in a park that starts with $\mathbf{10}$ rabbits and can sustain up to $\mathbf{60}$ rabbits. An equation that roughly models this scenario is

$$P = \frac{60}{1 + \frac{5}{t+1}},$$

where P represents the rabbit population in year t of the study.

What is the rabbit population in year $\mathbf{10}$? Round your answer to the nearest whole rabbit.

When asked about the rabbit population given the time, the problem is not very difficult although it looks complicated. We just have to be sure to find common denominators to deal with the complex fraction. Your calculator can easily handle this problem, if you correctly use parenthesis. But if we asked at what time does the population reach 50 rabbits, then the problem becomes much more difficult. For that we need some skill practice with rational expressions.

MULTIPLYING RATIONAL EXPRESSIONS

If a, b, c, and d are rational expressions with $b \neq 0$, $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}.$$

2. Make a conjecture about the product $\frac{x^3}{4y} \cdot \frac{y^2}{x}$. What will it be? Explain your conjecture, and give evidence that it is correct.

3. Find the following product:

$$\left(\frac{3x-6}{2x+6}\right) \cdot \left(\frac{5x+15}{4x+8}\right).$$

Instead of multiplying the binomials, factor first and see if you can simplify the expression.

4. Summarize what you have learned so far with your neighbor.

5. Find the following product and reduce to lowest terms: $\left(\frac{2x+6}{x^2+x-6}\right) \cdot \left(\frac{x^2-4}{2x}\right)$.

6. Find the following product and reduce to lowest terms: $\left(\frac{4n-12}{3m+6}\right)^{-2} \cdot \left(\frac{n^2-2n-3}{m^2+4m+4}\right)$.

DIVIDING RATIONAL EXPRESSIONS

If a, b, c, and d are rational expressions with $b \neq 0$, $c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

7. Find the quotient and reduce to lowest terms: $\frac{x^2-4}{3x} \div \frac{x-2}{2x}$.

Don't forget to factor!

8. Find the quotient and reduce to lowest terms: $\frac{x^2-5x+6}{x+4} \div \frac{x^2-9}{x^2+5x+4}$.

9. Simplify the rational expression.

$$\frac{\left(\frac{x+2}{x^2-2x-3}\right)}{\left(\frac{x^2-x-6}{x^2+6x+5}\right)}$$

Remember that the fraction bar represents division. You can change this complex fraction into a division problem.

Lesson Summary

In this lesson, we extended multiplication and division of rational numbers to multiplication and division of rational expressions.

- To multiply two rational expressions, multiply the numerators together and multiply the denominators together, and then reduce to lowest terms.
- To divide one rational expression by another, multiply the first by the multiplicative inverse of the second, and reduce to lowest terms.
- To simplify a complex fraction, apply the process for dividing one rational expression by another.

10. Kim wanted to give an example of dividing two rational expression but she made a mistake. Identify where she made her mistake and then correct it. Finish the problem for Kim.

$$\frac{x^2+x-6}{x^2+2x-3} \div \frac{x^2-3x+2}{x^2+6x+9} =$$

$$\frac{(x-2)(x+3)}{(x-1)(x+3)} \bullet \frac{(x-2)(x-1)}{(x+3)(x+3)} =$$

$$\frac{(x-2)\cancel{(x+3)}}{\cancel{(x-1)}\cancel{(x+3)}} \bullet \frac{(x-2)\cancel{(x-1)}}{(x+3)(x+3)} =$$

$$\frac{(x-2)^2}{(x+3)^2} = \frac{x^2-4x+4}{x^2+6x+9}$$

Homework Problem Set

1. Perform the following operations.

   A. Multiply $\frac{1}{3}(x-2)$ by 9.

   B. Divide $\frac{1}{4}(x-8)$ by $\frac{1}{12}$.

   C. Multiply $\frac{1}{4}\left(\frac{1}{3}x+2\right)$ by 12.

   D. Divide $\frac{1}{3}\left(\frac{2}{5}x-\frac{1}{5}\right)$ by $\frac{1}{15}$.

   E. Multiply $\frac{2}{3}\left(2x+\frac{2}{3}\right)$ by $\frac{9}{4}$.

   F. Multiply $0.03(4-x)$ by 100.

2. Write each rational expression as an equivalent rational expression in lowest terms.

A. $\left(\frac{a^3b^2}{c^2d^2}\cdot\frac{c}{ab}\right)\div\frac{a}{c^2d^3}$	B. $\frac{a^2+6a+9}{a^2-9}\cdot\frac{3a-9}{a+3}$

C. $\frac{6x}{4x-16} \div \frac{4x}{x^2-16}$	D. $\frac{3x^2-6x}{3x+1} \cdot \frac{x+3x^2}{x^2-4x+4}$
E. $\frac{y^2-x^2}{x+y} \cdot \frac{4}{2x-2y}$	F. $\frac{a-2b}{a+2b} \div (4b^2-a^2)$
G. $\frac{d+c}{c^2+d^2} \div \frac{c^2-d^2}{d^2-dc}$	H. $\frac{12a^2-7ab+b^2}{9a^2-b^2} \div \frac{16a^2-b^2}{3ab+b^2}$
I. $\left(\frac{x-3}{x^2-4}\right)^{-1} \cdot \left(\frac{x^2-x-6}{x-2}\right)$	J. $\left(\frac{x-2}{x^2+1}\right)^{-3} \div \left(\frac{x^2-4x+4}{x^2-2x-3}\right)$

K. $\frac{6x^2-11x-10}{6x^2-5x-6} \cdot \frac{6-4x}{25-20x+4x^2}$	L. $\frac{3x^3-3a^2x}{x^2-2ax+a^2} \cdot \frac{a-x}{a^3x+a^2x^2}$

3. Write each rational expression as an equivalent rational expression in lowest terms.

A. $\frac{\left(\frac{4a}{6b^2}\right)}{\left(\frac{20a^3}{12b}\right)}$

B. $\frac{\left(\frac{x-2}{x^2-1}\right)}{\left(\frac{x^2-4}{x-6}\right)}$

C. $\frac{\left(\frac{x^2+2x-3}{x^2+3x-4}\right)}{\left(\frac{x^2+x-6}{x+4}\right)}$

4. Suppose that $x = \frac{t^2+3t-4}{3t^2-3}$ and $y = \frac{t^2+2t-8}{2t^2-2t-4}$, for $t \neq 1$, $t \neq -1$, $t \neq 2$, and $t \neq -4$. Show that the value of x^2y^{-2} does not depend on the value of t.

Extension:

5. One of two numbers can be represented by the rational expression $\frac{x-2}{x}$, where $x \neq 0$ and $x \neq 2$.

   A. Find a representation of the second number if the product of the two numbers is 1.

   B. Find a representation of the second number if the product of the two numbers is 0.