Exit Ticket Sample Solutions

Perform the indicated operations, and reduce to lowest terms.

1. $\dfrac{\frac{x-2}{x^2+x-2}}{\frac{x^2-3x+2}{x+2}}$

$$\frac{x-2}{x^2+x-2}\cdot\frac{x^2-3x+2}{x+2}$$
$$=\frac{x-2}{(x-1)(x+2)}\cdot\frac{(x-1)(x-2)}{x+2}$$
$$=\frac{(x-2)^2}{(x+2)^2}$$

2. $\dfrac{\left(\frac{x-2}{x^2+x-2}\right)}{\left(\frac{x^2-3x+2}{x+2}\right)}$

$$\frac{\left(\frac{x-2}{x^2+x-2}\right)}{\left(\frac{x^2-3x+2}{x+2}\right)}=\frac{x-2}{x^2+x-2}$$
$$\div\frac{x^2-3x+2}{x+2}$$
$$=\frac{x-2}{x^2+x-2}\cdot\frac{x+2}{x^2-3x+2}$$
$$=\frac{x-2}{(x-1)(x+2)}$$
$$\cdot\frac{x+2}{(x-2)(x-1)}$$
$$=\frac{1}{(x-1)^2}$$

Homework Problem Set Sample Solutions

1. **Perform the following operations.**

a. **Multiply $\frac{1}{3}(x-2)$ by 9.**

$3x-6$

b. **Divide $\frac{1}{4}(x-8)$ by $\frac{1}{12}$.**

$3x-24$

c. **Multiply $\frac{1}{4}\left(\frac{1}{3}x+2\right)$ by 12.**

$x+6$

d. **Divide $\frac{1}{3}\left(\frac{2}{5}x-\frac{1}{5}\right)$ by $\frac{1}{15}$.**

$2x-1$

e. **Multiply $\frac{2}{3}\left(2x+\frac{2}{3}\right)$ by $\frac{9}{4}$.**

$3x+1$

f. **Multiply $0.03(4-x)$ by 100.**

$12-3x$

2. Write each rational expression as an equivalent rational expression in lowest terms.

a. $\left(\frac{a^3b^2}{c^2d^2} \cdot \frac{c}{ab}\right) \div \frac{a}{c^2d^3}$

$abcd$

b. $\frac{a^2+6a+9}{a^2-9} \cdot \frac{3a-9}{a+3}$

3

c. $\frac{6x}{4x-16} \div \frac{4x}{x^2-16}$

$\frac{3(x+4)}{8}$

d. $\frac{3x^2-6x}{3x+1} \cdot \frac{x+3x^2}{x^2-4x+4}$

$\frac{3x^2}{x-2}$

e. $\frac{y^2-x^2}{x+y} \cdot \frac{4}{2x-2y}$

-2

f. $\frac{a-2b}{a+2b} \div (4b^2-a^2)$

$-\frac{1}{(a+2b)^2}$

g. $\frac{d+c}{c^2+d^2} \div \frac{c^2-d^2}{d^2-dc}$

$-\frac{d}{c^2+d^2}$

h. $\frac{12a^2-7ab+b^2}{9a^2-b^2} \div \frac{16a^2-b^2}{3ab+b^2}$

$\frac{b}{4a+b}$

i. $\left(\frac{x-3}{x^2-4}\right)^{-1} \cdot \left(\frac{x^2-x-6}{x-2}\right)$

$(x+2)^2$

j. $\left(\frac{x-2}{x^2+1}\right)^{-3} \div \left(\frac{x^2-4x+4}{x^2-2x-3}\right)$

$\frac{(x-3)(x+1)(x^2+1)^3}{(x-2)^5}$

k. $\frac{6x^2-11x-10}{6x^2-5x-6} \cdot \frac{6-4x}{25-20x+4x^2}$

$-\frac{2}{2x-5}$, *or* $\frac{2}{5-2x}$

l. $\frac{3x^3-3a^2x}{x^2-2ax+a^2} \cdot \frac{a-x}{a^3x+a^2x^2}$

$-\frac{3}{a^2}$

3. Write each rational expression as an equivalent rational expression in lowest terms.

a. $\frac{\left(\frac{4a}{6b^2}\right)}{\left(\frac{20a^3}{12b}\right)}$

$\frac{2}{5a^2b}$

b. $\frac{\left(\frac{x-2}{x^2-1}\right)}{\left(\frac{x^2-4}{x-6}\right)}$

$\frac{x-6}{(x+2)(x^2-1)}$

c. $\frac{\left(\frac{x^2+2x-3}{x^2+3x-4}\right)}{\left(\frac{x^2+x-6}{x+4}\right)}$

$\frac{1}{x-2}$

4. Suppose that $x = \frac{t^2+3t-4}{3t^2-3}$ and $y = \frac{t^2+2t-8}{2t^2-2t-4}$, for $t \neq 1$, $t \neq -1$, $t \neq 2$, and $t \neq -4$. Show that the value of x^2y^{-2} does not depend on the value of t.

$$\begin{aligned} x^2y^{-2} &= \left(\frac{t^2+3t-4}{3t^2-3}\right)^2 \left(\frac{t^2+2t-8}{2t^2-2t-4}\right)^{-2} \\ &= \left(\frac{t^2+3t-4}{3t^2-3}\right)^2 \div \left(\frac{t^2+2t-8}{2t^2-2t-4}\right)^2 \\ &= \left(\frac{t^2+3t-4}{3t^2-3}\right)^2 \left(\frac{2t^2-2t-4}{t^2+2t-8}\right)^2 \\ &= \left(\frac{(t-1)(t+4)}{3(t-1)(t+1)}\right)^2 \left(\frac{2(t-2)(t+1)}{(t-2)(t+4)}\right)^2 \\ &= \frac{4(t-1)^2(t+4)^2(t-2)^2(t+1)^2}{9(t-1)^2(t+1)^2(t-2)^2(t+4)^2} \\ &= \frac{4}{9} \end{aligned}$$

Since $x^2y^{-2} = \frac{4}{9}$, the value of x^2y^{-2} does not depend on t.

Extension:

5. One of two numbers can be represented by the rational expression $\frac{x-2}{x}$, where $x \neq 0$ and $x \neq 2$.

   a. Find a representation of the second number if the product of the two numbers is 1.

   Let the second number be y. Then $\left(\frac{x-2}{x}\right) \cdot y = 1$, so we have

$$\begin{aligned} y &= 1 \div \left(\frac{x-2}{x}\right) \\ &= 1 \cdot \left(\frac{x}{x-2}\right) \\ &= \frac{x}{x-2}. \end{aligned}$$

   b. Find a representation of the second number if the product of the two numbers is 0.

   Let the second number be z. Then $\left(\frac{x-2}{x}\right) \cdot z = 0$, so we have

$$\begin{aligned} z &= 0 \div \left(\frac{x-2}{x}\right) = 0 \cdot \left(\frac{x}{x-2}\right) \\ &= 0. \end{aligned}$$