Exit Ticket Sample Solutions

Perform the indicated operation.

1. $\frac{3}{a+2}+\frac{4}{a-5}$

$$\frac{3}{a+2}+\frac{4}{a-5}=\frac{3a-15}{(a+2)(a-5)}+\frac{4a+8}{(a+2)(a-5)}$$
$$=\frac{7a-7}{(a+2)(a-5)}$$

2. $\frac{4r}{r+3}-\frac{5}{r}$

$$\frac{4r}{r+3}-\frac{5}{r}=\frac{4r^2}{r(r+3)}-\frac{5r+15}{r(r+3)}$$
$$=\frac{4r^2-5r-15}{r(r+3)}$$

Homework Problem Set Sample Solutions

1. **Write each sum or difference as a single rational expression.**

   a. $\frac{7}{8}-\frac{\sqrt{3}}{5}$

   $$\frac{35-8\sqrt{3}}{40}$$

   b. $\frac{\sqrt{5}}{10}+\frac{\sqrt{2}}{6}+2$

   $$\frac{3\sqrt{5}+5\sqrt{2}+60}{30}$$

   c. $\frac{4}{x}+\frac{3}{2x}$

   $$\frac{11}{2x}$$

2. **Write as a single rational expression.**

a. $\frac{1}{x} - \frac{1}{x-1}$

$-\frac{1}{x(x-1)}$

b. $\frac{3x}{2y} - \frac{5x}{6y} + \frac{x}{3y}$

$\frac{x}{y}$

c. $\frac{a-b}{a^2} + \frac{1}{a}$

$\frac{2a-b}{a^2}$

d. $\frac{1}{p-2} - \frac{1}{p+2}$

$\frac{4}{(p-2)(p+2)}$

e. $\frac{1}{p-2} + \frac{1}{2-p}$

0

f. $\frac{1}{b+1} - \frac{b}{1+b}$

$\frac{1-b}{b+1}$

g. $1 - \frac{1}{1+p}$

$\frac{p}{1+p}$

h. $\frac{p+q}{p-q} - 2$

$\frac{3q-p}{p-q}$

i. $\frac{r}{s-r} + \frac{s}{r+s}$

$\frac{r^2+s^2}{(s-r)(r+s)}$

j. $\frac{3}{x-4} + \frac{2}{4-x}$

$\frac{1}{x-4}$

k. $\frac{3n}{n-2} + \frac{3}{2-n}$

$\frac{3n-3}{n-2}$

l. $\frac{8x}{3y-2x} + \frac{12y}{2x-3y}$

-4

m. $\frac{1}{2m-4n} - \frac{1}{2m+4n} - \frac{m}{m^2-4n^2}$

$-\frac{1}{m+2n}$

n. $\frac{1}{(2a-b)(a-c)} + \frac{1}{(b-c)(b-2a)}$

$\frac{b-a}{(a-c)(b-c)(2a-b)}$

o. $\frac{b^2+1}{b^2-4} + \frac{1}{b+2} + \frac{1}{b-2}$

$\frac{b^2+2b+1}{(b-2)(b+2)}$

CHALLENGE PROBLEMS

3. **Write each rational expression as an equivalent rational expression in lowest terms.**

a. $\dfrac{\frac{1}{a}-\frac{1}{2a}}{\frac{4}{a}}$

$\dfrac{1}{8}$

b. $\dfrac{\frac{5x}{2}+1}{\frac{5x}{4}-\frac{1}{5x}}$

$\dfrac{10x}{5x-2}$

c. $\dfrac{1+\frac{4x+3}{x^2+1}}{1-\frac{x+7}{x^2+1}}$

$\dfrac{x+2}{x-3}$

Extension:

4. **Suppose that $x \neq 0$ and $y \neq 0$. We know from our work in this section that $\frac{1}{x}\cdot\frac{1}{y}$ is equivalent to $\frac{1}{xy}$. Is it also true that $\frac{1}{x}+\frac{1}{y}$ is equivalent to $\frac{1}{x+y}$? Provide evidence to support your answer.**

No, the rational expressions $\frac{1}{x}+\frac{1}{y}$ and $\frac{1}{x+y}$ are not equivalent. Consider $x = 2$ and $y = 1$. Then $\frac{1}{x+y} = \frac{1}{2+1} = \frac{1}{3}$, but $\frac{1}{x}+\frac{1}{y} = \frac{1}{2}+1 = \frac{3}{2}$. Since $\frac{1}{3} \neq \frac{3}{2}$, the expressions $\frac{1}{x}+\frac{1}{y}$ and $\frac{1}{x+y}$ are not equivalent.

5. **Suppose that $x = \frac{2t}{1+t^2}$ and $y = \frac{1-t^2}{1+t^2}$. Show that the value of x^2+y^2 does not depend on the value of t.**

$$\begin{aligned}
x^2+y^2 &= \left(\frac{2t}{1+t^2}\right)^2 + \left(\frac{1-t^2}{1+t^2}\right)^2 \\
&= \frac{4t^2}{(1+t^2)^2} + \frac{(1-t^2)^2}{(1+t^2)^2} \\
&= \frac{4t^2+(1-2t^2+t^4)}{(1+t^2)^2} \\
&= \frac{1+2t^2+t^4}{1+2t^2+t^4} \\
&= 1
\end{aligned}$$

Since $x^2+y^2 = 1$, the value of x^2+y^2 does not depend on the value of t.

6. **Show that for any real numbers a and b, and any integers x and y so that $x \neq 0$, $y \neq 0$, $x \neq y$, and $x \neq -y$,**

$$\left(\frac{y}{x} - \frac{x}{y}\right)\left(\frac{ax+by}{x+y} - \frac{ax-by}{x-y}\right) = 2(a-b).$$

$$\begin{aligned}
\left(\frac{y}{x} - \frac{x}{y}\right)\left(\frac{ax+by}{x+y} - \frac{ax-by}{x-y}\right) &= \left(\frac{y^2}{xy} - \frac{x^2}{xy}\right)\left(\frac{(ax+by)(x-y)}{(x+y)(x-y)} - \frac{(ax-by)(x+y)}{(x-y)(x+y)}\right) \\
&= \left(\frac{y^2 - x^2}{xy}\right)\left(\frac{ax^2 - axy + bxy - by^2}{x^2 - y^2} - \frac{(ax^2 + axy - bxy - by^2}{x^2 - y^2}\right) \\
&= -\left(\frac{x^2 - y^2}{xy}\right)\left(\frac{-2axy + 2bxy}{x^2 - y^2}\right) \\
&= -\left(\frac{1}{xy}\right)\left(\frac{-2xy(a-b)}{1}\right) \\
&= 2(a-b)
\end{aligned}$$

7. Suppose that n is a positive integer.

a. Rewrite the product in the form $\frac{P}{Q}$ for polynomials P and Q: $\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)$.

$$\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)=\left(\frac{n+1}{n}\right)\left(\frac{n+2}{n+1}\right)=\left(\frac{n+2}{n}\right)$$

b. Rewrite the product in the form $\frac{P}{Q}$ for polynomials P and Q: $\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)\left(1+\frac{1}{n+2}\right)$.

$$\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)\left(1+\frac{1}{n+2}\right)=\left(\frac{n+1}{n}\right)\left(\frac{n+2}{n+1}\right)\left(\frac{n+3}{n+2}\right)=\left(\frac{n+3}{n}\right)$$

c. Rewrite the product in the form $\frac{P}{Q}$ for polynomials P and Q: $\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)\left(1+\frac{1}{n+2}\right)\left(1+\frac{1}{n+3}\right)$.

$$\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)\left(1+\frac{1}{n+2}\right)\left(1+\frac{1}{n+3}\right)$$
$$=\left(\frac{n+1}{n}\right)\left(\frac{n+2}{n+1}\right)\left(\frac{n+3}{n+2}\right)\left(\frac{n+4}{n+3}\right)=\left(\frac{n+4}{n}\right)$$

MP.7

d. If this pattern continues, what is the product of n of these factors?

If we have n of these factors, then the product will be

$$\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)\cdots\left(1+\frac{1}{n+(n-1)}\right)=\frac{n+n}{n}=2.$$