Exit Ticket Sample Solutions

Find all solutions to the following equation. If there are any extraneous solutions, identify them and explain why they are extraneous.

$$\frac{7}{b+3}+\frac{5}{b-3}=\frac{10b}{b^2-9}$$

First, note that we must have $x \neq 3$ and $x \neq -3$.

Using the equating numerators method: $\frac{7(b-3)}{(b-3)(b+3)}+\frac{5(b+3)}{(b-3)(b+3)}=\frac{10b}{(b-3)(b+3)}$

Matching numerators, we have $7b-21+5b+15=10b$, which leads to $12b-6=10b$; therefore, $b=3$.

However, since the excluded values are 3 and -3, the solution 3 is an extraneous solution, and there is no solution to $\frac{7}{b+3}+\frac{5}{b-3}=\frac{10b}{b^2-9}$.

Homework Problem Set Sample Solutions

1. **Solve the following equations, and check for extraneous solutions.**

a. $\frac{x-8}{x-4}=2$

0

b. $\frac{4x-8}{x-2}=4$

All real numbers except 2

c. $\frac{x-4}{x-3}=1$

No solution

d. $\frac{4x-8}{x-2}=3$

No solution

e. $\frac{1}{2a}-\frac{2}{2a-3}=0$

$-\frac{3}{2}$

f. $\frac{3}{2x+1}=\frac{5}{4x+3}$

-2

g. $\frac{4}{x-5}-\frac{2}{5+x}=\frac{2}{x}$

$-\frac{5}{3}$

h. $\frac{y+2}{3y-2}+\frac{y}{y-1}=\frac{2}{3}$

$\frac{5}{6}, -2$

i. $\frac{4}{x-1}+\frac{3}{x}-3=0$

$\frac{1}{3}, 3$

j. $\frac{x+1}{x+3}-\frac{x-5}{x+2}=\frac{17}{6}$

$0, -\frac{55}{17}$

k. $\frac{3}{x+1}-\frac{2}{1-x}=1$

$0, 5$

l. $\frac{x+7}{4}-\frac{x+1}{2}=\frac{5-x}{3x-14}$

$5, 6$

m. $\frac{b^2-b-6}{b^2} - \frac{2b+12}{b} = \frac{b-39}{2b}$

$3, \frac{4}{3}$

n. $\frac{1}{p(p-4)} + 1 = \frac{p-6}{p}$

$\frac{23}{6}$

o. $\frac{1}{h+3} = \frac{h+4}{h-2} + \frac{6}{h-2}$

$-8, -4$

2. **Create and solve a rational equation that has 0 as an extraneous solution.**

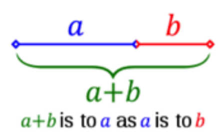
One such equation is $\frac{1}{x-1} + \frac{1}{x} = \frac{1}{x-x^2}$.

3. **Create and solve a rational equation that has 2 as an extraneous solution.**

One such equation is $\frac{1}{x-2} + \frac{1}{x+2} = \frac{4}{x^2-4}$.

Extension:

4. **Two lengths a and b, where $a > b$, are in *golden ratio* if the ratio of $a+b$ is to a is the same as a is to b. Symbolically, this is expressed as $\frac{a}{b} = \frac{a+b}{a}$. We denote this common ratio by the Greek letter *phi (pronounced "fee")* with symbol φ, so that if a and b are in common ratio, then $\varphi = \frac{a}{b} = \frac{a+b}{a}$. By setting $b = 1$, we find that $\varphi = a$ and φ is the positive number that satisfies the equation $\varphi = \frac{\varphi+1}{\varphi}$. Solve this equation to find the numerical value for φ.**



We can apply either method from the previous lesson to solve this equation.

$$\varphi = \frac{\varphi + 1}{\varphi}$$

$$\varphi^2 = \varphi + 1$$

$$\varphi^2 - \varphi - 1 = 0$$

Applying the quadratic formula, we have two solutions:

$$\varphi = \frac{1+\sqrt{5}}{2} \quad \textit{or} \quad \varphi = \frac{1-\sqrt{5}}{2}.$$

Since φ is a positive number, and $\frac{1-\sqrt{5}}{2} < 0$, *we have* $\varphi = \frac{1+\sqrt{5}}{2}$.

5. Remember that if we use x to represent an integer, then the next integer can be represented by $x+1$.

   a. Does there exist a pair of consecutive integers whose reciprocals sum to $\frac{5}{6}$? Explain how you know.

   Yes, 2 and 3 because $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

   b. Does there exist a pair of consecutive integers whose reciprocals sum to $\frac{3}{4}$? Explain how you know.

   If x represents the first integer, then $x+1$ represents the next integer. Suppose $\frac{1}{x}+\frac{1}{x+1}=\frac{3}{4}$. Then,

$$\frac{1}{x}+\frac{1}{x+1}=\frac{3}{4}$$
$$\frac{4(x+1)+4x}{4x(x+1)}=\frac{3x(x+1)}{4x(x+1)}$$
$$8x+4=3x^2+3x$$
$$3x^2-5x-4=0.$$

   The solutions to this quadratic equation are $\frac{5+\sqrt{73}}{6}$ and $\frac{5-\sqrt{73}}{6}$, so there are no integers that solve this equation. Thus, there are no pairs of consecutive integers whose reciprocals sum to $\frac{3}{4}$.

c. **Does there exist a pair of consecutive *even* integers whose reciprocals sum to $\frac{3}{4}$? Explain how you know.**

If x represents the first integer, then $x+2$ represents the next even integer. Suppose $\frac{1}{x}+\frac{1}{x+2}=\frac{3}{4}$. Then,

$$\frac{1}{x}+\frac{1}{x+2}=\frac{3}{4}$$
$$\frac{4(x+2)+4x}{4x(x+2)}=\frac{3x(x+2)}{4x(x+2)}$$
$$8x+8=3x^2+6x$$
$$3x^2-2x-8=0.$$

The solutions to this quadratic equation are $-\frac{4}{3}$ and 2; therefore, the only even integer x that solves the equation is 2. Then, 2 and 4 are consecutive even integers whose reciprocals sum to $\frac{3}{4}$.

d. **Does there exist a pair of consecutive *even* integers whose reciprocals sum to $\frac{5}{6}$? Explain how you know.**

If x represents the first integer, then $x+2$ represents the next even integer. Suppose $\frac{1}{x}+\frac{1}{x+2}=\frac{5}{6}$. Then,

$$\frac{1}{x}+\frac{1}{x+2}=\frac{5}{6}$$
$$\frac{6(x+2)+6x}{6x(x+2)}=\frac{5x(x+2)}{6x(x+2)}$$
$$12x+12=5x^2+10x$$
$$5x^2-2x-12=0.$$

The solutions to this quadratic equation are $\frac{1+\sqrt{61}}{5}$ and $\frac{1-\sqrt{61}}{5}$, so there are no integers that solve this equation. Thus, there are no pairs of consecutive even integers whose reciprocals sum to $\frac{5}{6}$.