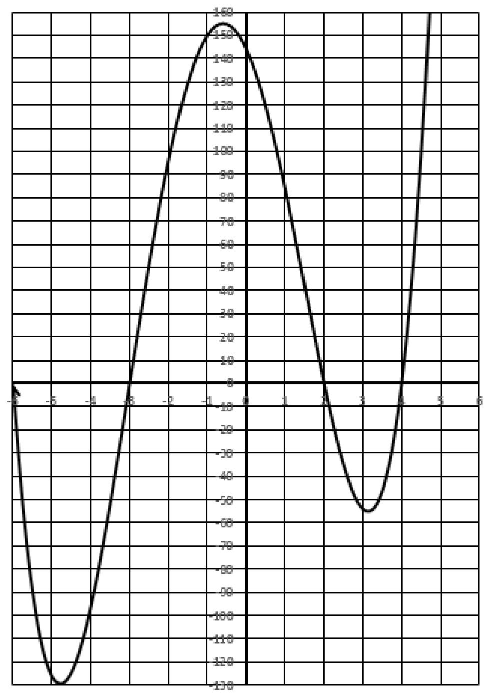
Lesson 21: Polynomials Revisited: The Remainder Theorem

Exploratory Exercise

1. Consider the polynomial $P(x) = x^4 + 3x^3 - 28x^2 - 36x + 144$ shown graphed on the right.



A. Is 1 a zero of the polynomial P? How do you know?

B. Divide $P(x)$ by $x - 1$. You may use any method (long division, reserve tabular method or synthetic division). What is the remainder?

C. What is $f(1)$? How does this answer compare to the remainder in Part B?

D. Is $x + 3$ one of the factors of P? How do you know?

E. Divide $P(x)$ by $x + 3$. What is the remainder?

F. What is $f(-3)$? How does this answer compare to the remainder in Part E?

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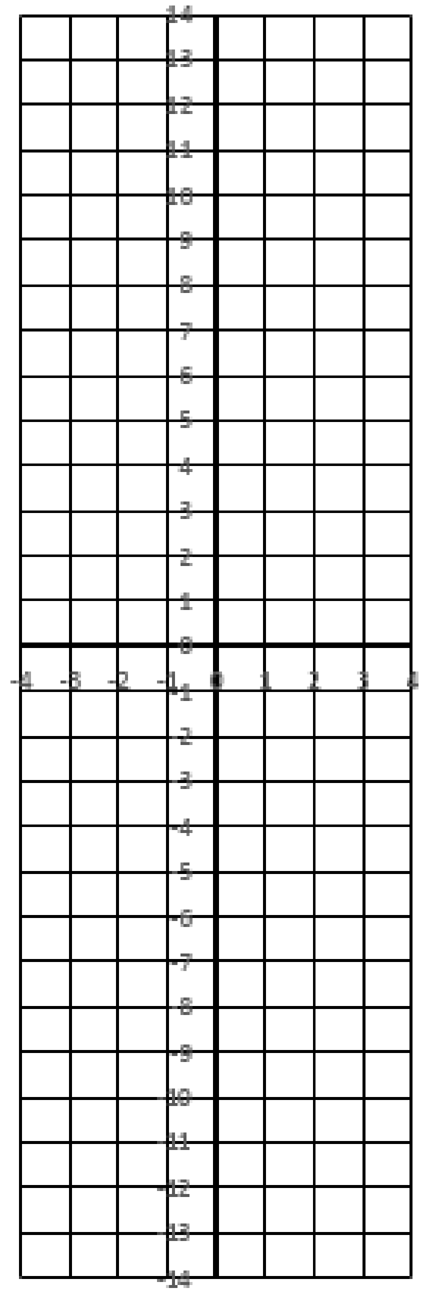
2. The polynomial function, $f(x) = x^4 + x^3 - 7x^2 - x + 6$, has a zero at -3. Graph the function on the grid at the right.



   A. Use what you learned in Exercise 1 and your knowledge of polynomials from Unit 1 to create an accurate graph of the function.

   B. What is the value of $f(-2)$? Mark this point on your graph.

   C. What is the value of $f(0)$? Mark this point on your graph.

   D. What is the value of $f(-4)$? Can this be marked on your graph?

Discussion

3. The Remainder Theorem states: If the polynomial $P(x)$ is divided by $x - a$, then the remainder is ________.

4. Why is this important? How this can help us graph polynomial functions?

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5. Consider the polynomial function $f(x) = 3x^2 + 8x - 4$.

   A. Divide f by $x - 2$.

   B. Find $f(2)$.

6. Consider the polynomial function $g(x) = x^3 - 3x^2 + 6x + 8$.

   A. Divide g by $x + 1$.

   B. Find $g(-1)$.

7. Consider the polynomial function $h(x) = x^3 + 2x - 3$.

   A. Divide h by $x - 3$.

   B. Find $h(3)$.

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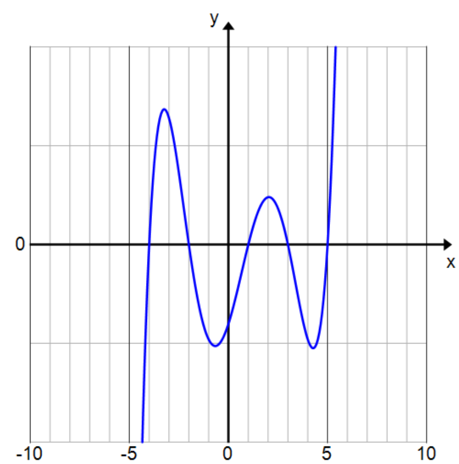
8. Consider the polynomial $P(x) = x^3 + kx^2 + x + 6$.

   A. Find the value of k so that $x + 1$ is a factor of P.

   B. Find the other two factors of P for the value of k found in Part A

9. Consider the graph of a degree 5 polynomial shown to the right, with x-intercepts -4, -2, 1, 3, and 5.

   A. Write a formula for a possible polynomial function that the graph represents using c as the constant factor.



   B. Suppose the y-intercept is -4. Find the value of c so that the graph of P has y-intercept -4.

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Lesson Summary

REMAINDER THEOREM: Let P be a polynomial function in x, and let a be any real number. Then there exists a unique polynomial function q such that the equation

$$P(x) = q(x)(x - a) + P(a)$$

is true for all x. That is, when a polynomial is divided by $(x - a)$, the remainder is the value of the polynomial evaluated at a.

FACTOR THEOREM: Let P be a polynomial function in x, and let a be any real number. If a is a zero of P, then $(x - a)$ is a factor of P.

Example: If $P(x) = x^3 - 5x^2 + 3x + 9$, then $P(3) = 27 - 45 + 9 + 9 = 0$, so $(x - 3)$ is a factor of P.

Homework Problem Set

1. Use the remainder theorem to find the remainder for each of the following divisions.

A. $\frac{(x^2+3x+1)}{(x+2)}$

B. $\frac{x^3-6x^2-7x+9}{(x-3)}$

C. $\frac{32x^4+24x^3-12x^2+2x+1}{(x+1)}$

D. $\frac{32x^4+24x^3-12x^2+2x+1}{(2x-1)}$

Hint: Can you rewrite the division expression so that the divisor is in the form $(x - c)$ for some constant c?

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2. Consider the polynomial $P(x) = x^3 + 6x^2 - 8x - 1$. Find $P(4)$ in two ways.

3. Consider the polynomial function $P(x) = 2x^4 + 3x^2 + 12$.
   a. Divide P by $x + 2$, and rewrite P in the form (divisor)(quotient) + remainder.

   b. Find $P(-2)$.

4. Consider the polynomial function $P(x) = x^3 + 42$.
   a. Divide P by $x - 4$, and rewrite P in the form (divisor)(quotient)+remainder.

   b. Find $P(4)$.

5. For a polynomial function P, $P(a)$ is equal to the remainder of the quotient of P and $x - a$. Why is this true?

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6. Is $x - 5$ a factor of the function $f(x) = x^3 + x^2 - 27x - 15$? Show work supporting your answer.

7. Is $x + 1$ a factor of the function $f(x) = 2x^5 - 4x^4 + 9x^3 - x + 13$? Show work supporting your answer.

8. A polynomial function p has zeros of $2, 2, -3, -3, -3$, and 4. Find a possible formula for P, and state its degree. Why is the degree of the polynomial not 3?

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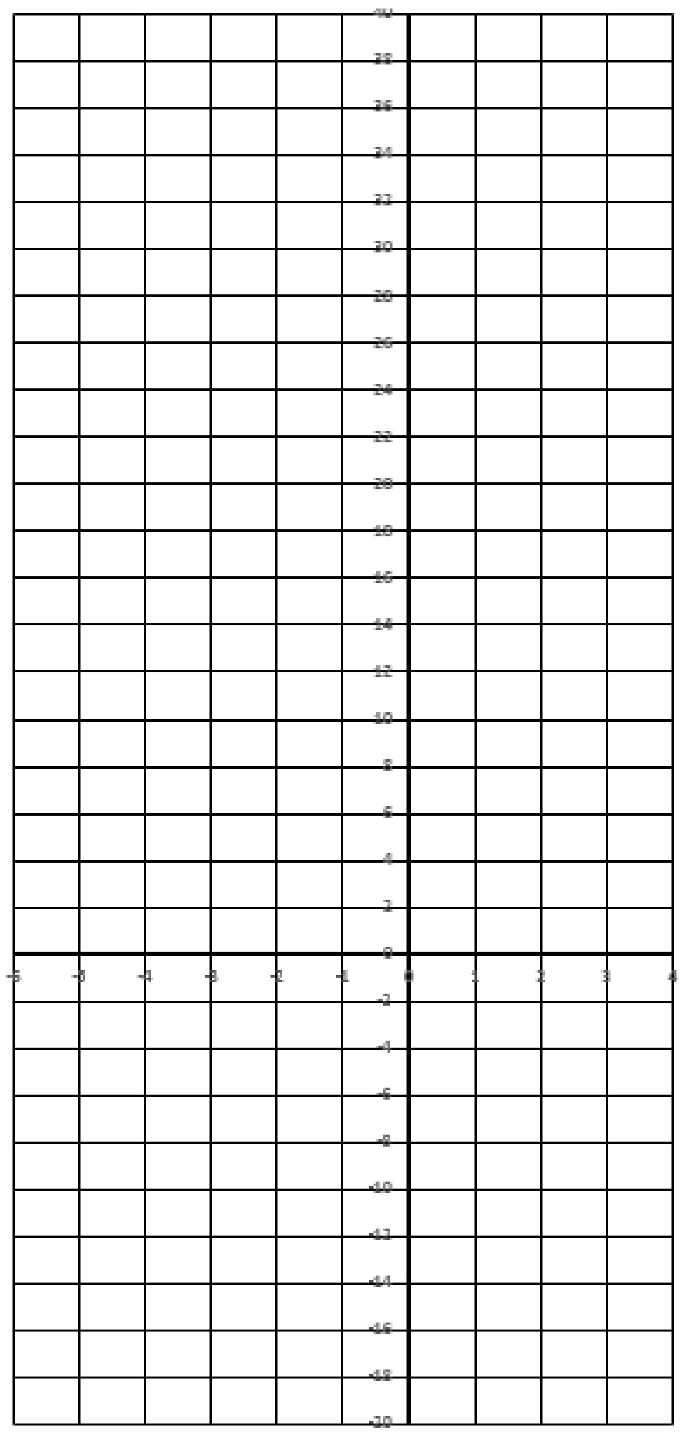
9. Consider the polynomial function $P(x) = x^3 + 2x^2 - 15x$.

   a. Verify that $P(0) = 0$. Since $P(0) = 0$, what must one of the factors of P be?

   b. Find the remaining two factors of P.

   c. State the zeros of P.

   d. Sketch the graph of P.



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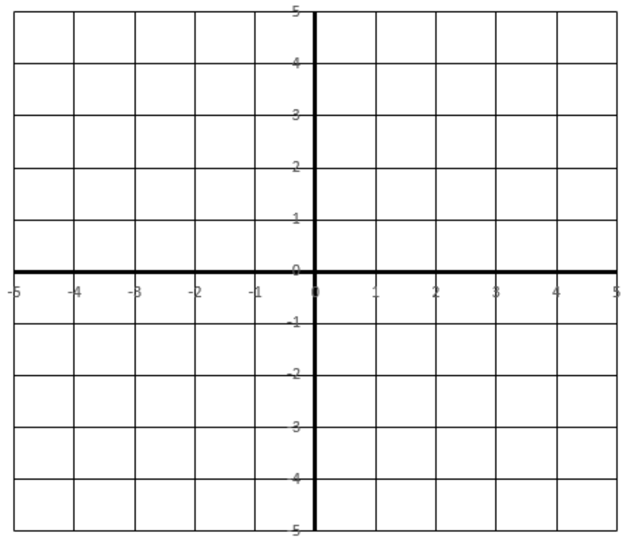
10. Consider the polynomial function $P(x) = 2x^3 + 3x^2 - 2x - 3$.
   a. Verify that $P(-1) = 0$. Since $P(-1) = 0$, what must one of the factors of P be?

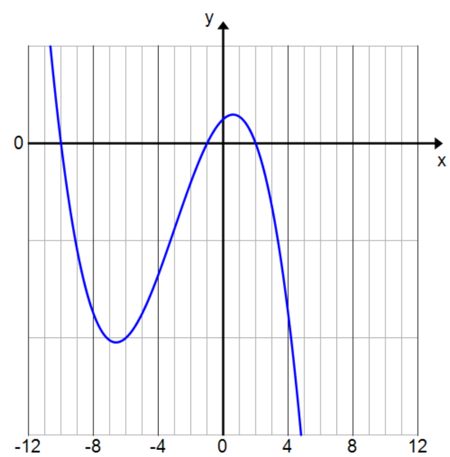


   b. Find the remaining two factors of P.

   c. State the zeros of P.

   d. Sketch the graph of P.

11. The graph to the right is of a third-degree polynomial function f.



   a. State the zeros of f.

   b. Write a formula for f in factored form using c for the constant factor.

   c. Use the fact that $f(-4) = -54$ to find the constant factor c.

   d. Verify your equation by using the fact that $f(1) = 11$.

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12. Find the value of k so that $\frac{x^3-kx^2+2}{x-1}$ has remainder 8.

13. Find the value k so that $\frac{kx^3+x-k}{x+2}$ has remainder 16.

14. Show that $x^{51} - 21x + 20$ is divisible by $x - 1$.

15. Show that $x + 1$ is a factor of $19x^{42} + 18x - 1$.

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