Exit Ticket Sample Solutions

Consider polynomial $P(x) = x^3 + x^2 - 10x - 10$.

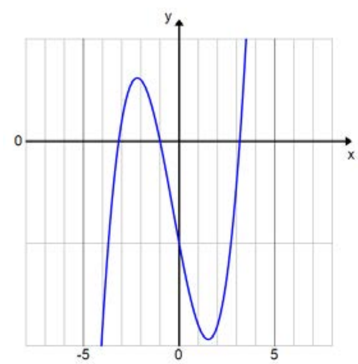
1. Is $x + 1$ one of the factors of P? Explain.

   $P(-1) = (-1)^3 + (-1)^2 - 10(-1) - 10 = -1 + 1 + 10 - 10 = 0$

   Yes, $x + 1$ is a factor of P because $P(-1) = 0$. Or, using factoring by grouping, we have

   $P(x) = x^2(x + 1) - 10(x + 1) = (x + 1)(x^2 - 10)$.

2. The graph shown has x-intercepts at $\sqrt{10}$, -1, and $-\sqrt{10}$. Could this be the graph of $P(x) = x^3 + x^2 - 10x - 10$? Explain how you know.



   Yes, this could be the graph of P. Since this graph has x-intercepts at $\sqrt{10}$, -1, and $-\sqrt{10}$, the factor theorem says that $(x - \sqrt{10})$, $(x - 1)$, and $(x + \sqrt{10})$ are all factors of the equation that goes with this graph. Since $(x - \sqrt{10})(x + \sqrt{10})(x - 1) = x^3 + x^2 - 10x - 10$, the graph shown is quite likely to be the graph of P.

Homework Problem Set Sample Solutions

1. Use the remainder theorem to find the remainder for each of the following divisions.

   a. $\frac{(x^2+3x+1)}{(x+2)}$

   -1

   b. $\frac{x^3-6x^2-7x+9}{(x-3)}$

   -39

   c. $\frac{32x^4+24x^3-12x^2+2x+1}{(x+1)}$

   -5

   d. $\frac{32x^4+24x^3-12x^2+2x+1}{(2x-1)}$

   Hint for part (d): Can you rewrite the division expression so that the divisor is in the form $(x - c)$ for some constant c?

   4

2. **Consider the polynomial $P(x) = x^3 + 6x^2 - 8x - 1$. Find $P(4)$ in two ways.**

$P(4) = 4^3 + 6(4)^2 - 8(4) - 1 = 127$

$\frac{x^3+6x^2-8x-1}{x-4}$ *has a remainder of* 127, *so* $P(4) = 127$.

3. **Consider the polynomial function $P(x) = 2x^4 + 3x^2 + 12$.**
   a. **Divide P by $x + 2$, and rewrite P in the form (divisor)(quotient)+remainder.**

      $P(x) = (x + 2)(2x^3 - 4x^2 + 11x - 22) + 56$
   b. **Find $P(-2)$.**

      $P(-2) = (-2 + 2)\big(q(-2)\big) + 56 = 56$

4. **Consider the polynomial function $P(x) = x^3 + 42$.**
   a. **Divide P by $x - 4$, and rewrite P in the form (divisor)(quotient) + remainder.**

      $P(x) = (x - 4)(x^2 + 4x + 16) + 106$
   b. **Find $P(4)$.**

      $P(4) = (4 - 4)\big(q(4)\big) + 106 = 106$

5. **Explain why for a polynomial function P, $P(a)$ is equal to the remainder of the quotient of P and $x - a$.**

   The polynomial P can be rewritten in the form $\mathrm{P(x) = (x - a)(q(x)) + r}$, where $\mathrm{q(x)}$ is the quotient function and
   r is the remainder. Then $\mathrm{P(a) = (a - a)\big(q(a)\big) + r}$. Therefore, $\mathrm{P(a) = r}$.

6. **Is $x - 5$ a factor of the function $f(x) = x^3 + x^2 - 27x - 15$? Show work supporting your answer.**

   Yes, because $\mathrm{f(5) = 0}$ means that dividing by $\mathrm{x - 5}$ leaves a remainder of 0.

7. **Is $x+1$ a factor of the function $f(x) = 2x^5 - 4x^4 + 9x^3 - x + 13$? Show work supporting your answer.**

No, because $f(-1) = -1$ means that dividing by $x + 1$ has a remainder of -1.

8. **A polynomial function p has zeros of $2, 2, -3, -3, -3$, and 4. Find a possible formula for P, and state its degree. Why is the degree of the polynomial not 3?**

One solution is $P(x) = (x-2)^2(x+3)^3(x-4)$. The degree of P is 6. This is not a degree 3 polynomial function because the factor $(x-2)$ appears twice, and the factor $(x+3)$ appears 3 times, while the factor $(x-4)$ appears once.

9. **Consider the polynomial function $P(x) = x^3 + 2x^2 - 15x$.**

   a. **Verify that $P(0) = 0$. Since $P(0) = 0$, what must one of the factors of P be?**

   $P(0) = 90^3 + 2(0^2) - 15(0)0 = 0$; $x - 0$ or x.

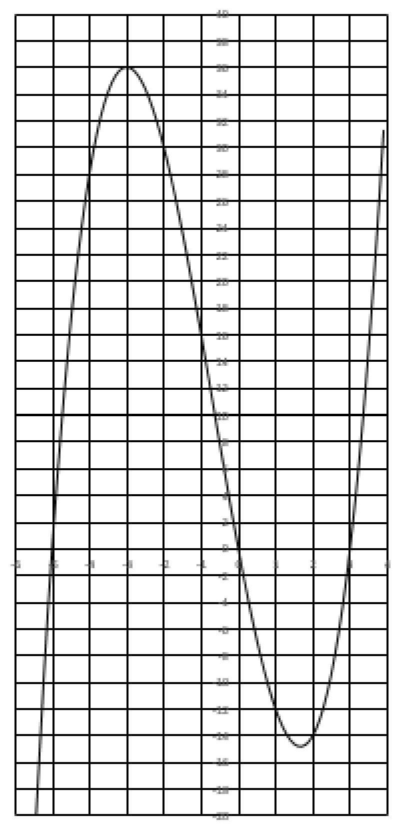
   b. **Find the remaining two factors of P.**

   $P(x) = x(x-3)(x+5)$

   c. **State the zeros of P.**

   $x = 0, 3, -5$

   d. **Sketch the graph of P.**



10. **Consider the polynomial function $P(x) = 2x^3 + 3x^2 - 2x - 3$.**

a. **Verify that $P(-1) = 0$. Since $P(-1) = 0$, what must one of the factors of P be?**

$P(-1) = 2(-1)^3 + 3(-1)^2 - 2(-1) - 3 = 0;\ \ x + 1$

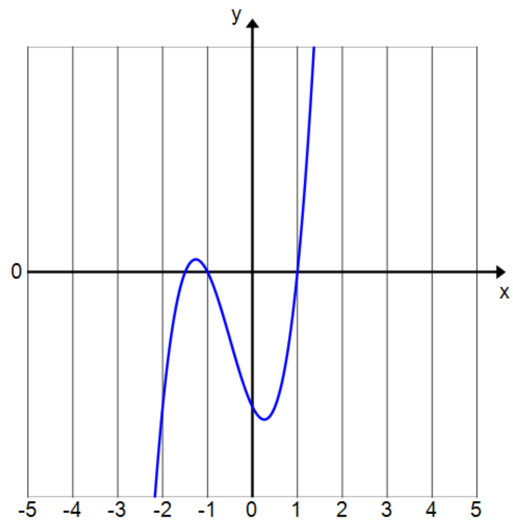
b. **Find the remaining two factors of P.**

$P(x) = (x + 1)(x - 1)(2x + 3)$

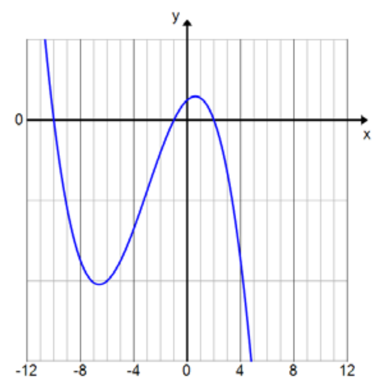
c. **State the zeros of P.**

$x = -1,\ 1,\ -\frac{3}{2}$

d. **Sketch the graph of P.**



11. **The graph to the right is of a third-degree polynomial function f.**



a. **State the zeros of f.**

$x = -10,\ -1,\ 2$

b. **Write a formula for f in factored form using c for the constant factor.**

$f(x) = c(x + 10)(x + 1)(x - 2)$

c. **Use the fact that $f(-4) = -54$ to find the constant factor c.**

$-54 = c(-4 + 10)(-4 + 1)(-4 - 2)$

$c = -\frac{1}{2}$

$f(x) = -\frac{1}{2}(x + 10)(x + 1)(x - 2)$

d. **Verify your equation by using the fact that $f(1) = 11$.**

$f(1) = -\frac{1}{2}(1 + 10)(1 + 1)(1 - 2) = -\frac{1}{2}(11)(2)(-1) = 11$

12. **Find the value of k so that $\frac{x^3-kx^2+2}{x-1}$ has remainder 8.**

$k = -5$

13. **Find the value k so that $\frac{kx^3+x-k}{x+2}$ has remainder 16.**

$k = -2$

14. **Show that $x^{51} - 21x + 20$ is divisible by $x - 1$.**

Let $P(x) = x^{51} - 21x + 20$.

Then $P(1) = 1^{51} - 21(1) + 20 = 1 - 21 + 20 = 0$.

Since $P(1) = 0$, *the remainder of the quotient* $(x^{51} - 21x + 20) \div (x - 1)$ *is* 0.

Therefore, $x^{51} - 21x + 20$ *is divisible by* $x - 1$.

15. **Show that $x + 1$ is a factor of $19x^{42} + 18x - 1$.**

Let $P(x) = 19x^{42} + 18x - 1$.

Then $P(-1) = 19(-1)^{42} + 18(-1) - 1 = 19 - 18 - 1 = 0$.

Since $P(-1) = 0$, $x + 1$ *must be a factor of* P.