Lesson 22: Think Fast! – A Focus on Roots

Exploratory Activity

Have you ever had someone shout, “Think fast!” and then thrown something at you? Do you usually catch it? How fast do you have to be to be a fighter pilot or race car driver? Does playing video games make your reaction time faster? In the next activity, you and your partner will test your reaction time.

You and your partner will need: one centimeter ruler

1. Follow the directions below to test your reaction time.

- Sit in a chair with your arm resting on a table so that your wrist hangs off the edge.
- Your partner will hold the ruler just above your hand so that the ruler will fall between your thumb and forefinger. Your thumb and forefinger should be about 8 centimeters apart. Be sure the 0 centimeter mark is closest to your hand.
- When your partner lets go of the ruler, try to catch it between your thumb and forefinger as quickly as you can. (Your partner should drop the ruler without any warning, so it is important that your partner say anything to warn you and the wait time should vary with each trial.)
- Record the number of centimeters where your fingers caught the ruler.
- Do this test three times and then have your partner try to catch the ruler three times.
- Use the chart below to see what your reaction time is.

My reaction times:

_______, _______, _______

Reaction Time Chart

Distance on Ruler (centimeter)	Reaction Time (milliseconds)	Rating/Comment	Distance on Ruler (centimeter)	Reaction Time (milliseconds)	Rating/Comment
1	50	Ultra-fast - Are you cheating??	16	180	Good - Keep trying! You are not a top gun yet.
2	60	Superb - Impressive! Do you play computer games?	17	190	
3	70		18	190	
4	80		19	200	
5	90		20	200	
6	100		21	210	Average - Would you be faster if it were money instead of a ruler?
7	120		22	210	
8	130		23	220	
9	140	Excellent - Well done!	24	220	
10	140		25	230	
11	150		26	230	
12	160		27	230	
13	160		28	240	
14	170		29	240	
15	170		30	250	Ouch! Did the ruler hit your foot?

The reaction times shown in the table were determined using the formula $d = \frac{1}{2} \bullet 980 \bullet t^2$, where d= distance in centimeters the ruler falls, t = reaction time in seconds, 980 cm/sec ^{2} is the acceleration of a falling mass on Earth.

2. Let's see how accurate those times are.

   A. If the reaction time was 50 milliseconds, how many seconds would this be? There are 1000 milliseconds in 1 second.

   B. Use the formula to determine the distance the ruler fell with your answer in Part A. How close is this to the number in the table?

   C. If the reaction time was 170 milliseconds, what is the distance the ruler fell? (Be sure to change milliseconds to seconds.) How close is this to the value in the table?

   D. Why do you think the numbers are not exactly the same?

3. Suppose you wanted to know the reaction time when the ruler falls 20.5 centimeters.

   A. Write the equation you could use.

   B. How could you isolate the variable t?

   C. Rewrite the equation $d = \frac{1}{2} \bullet 980 \bullet t^2$ and solve for t.

   D. Why wouldn't we use ± in Part C?

Practice Solving Radical Equations

Describe each step taken to solve the equation. Then, check the solution to see if it is valid. If it is not a valid solution, explain why.

4. $\sqrt{x} - 6 = 4$

   $\sqrt{x} = 10$

   $x = 100$

5. $\sqrt[3]{x} - 6 = 4$

   $\sqrt[3]{x} = 10$

   $x = 1000$

6. $\sqrt{x} + 6 = 4$

   $\sqrt{x} = -2$

   $x = 4$

7. $\sqrt[3]{x} + 6 = 4$

   $\sqrt[3]{x} = -2$

   $x = -8$

Discussion

8. A. What was the first step taken in solving the radical equations in Exercises 4 – 7?

   B. What was the second step taken?

   C. What happened in Exercise 6? Why did this happen?

9. Solve the following radical equation. Be sure to check your solutions.

$$\sqrt{3x+5}-2=-1$$

10. Solve each radical equation. Be sure to check your solutions.

A. $\sqrt{2x-3}=11$

B. $\sqrt[3]{6-x}=-3$

C. $\sqrt{x+5}-9=-12$

D. $\sqrt{4x-7}=\sqrt{3x+9}$

E. $-12\sqrt{x-6}=18$

F. $3\sqrt[3]{x+2}=12$

G. $\sqrt{x^2-5}=2$

H. $\sqrt{x^2+8x}=3$

11. A. Which Parts in the last exercise produced extraneous solutions?

B. Which Parts in the last exercise produced more than one solution? Why?

C. Write a racial equation that has an extraneous solution.

D. Exchange problems with your partner and confirm the extraneous solution.

Partner's Problem: ___________________________

Radical Multiplication

12. Compute each product, and combine like terms.

A. $\left(\sqrt{x}+2\right)\left(\sqrt{x}-2\right)$

B. $\left(\sqrt{x}+4\right)\left(\sqrt{x}+4\right)$

C. $\left(\sqrt{x-5}\right)\left(\sqrt{x-5}\right)$

Rationalizing Denominators

13. Rationalize the denominator in each expression. That is, rewrite the expression so that there is a rational expression in the denominator.

A. $\frac{x-9}{\sqrt{x-9}}$

B. $\frac{x-9}{\sqrt{x}+3}$

14. Rewrite $\frac{1}{\sqrt{x}-5}$ in an equivalent form with a rational expression in the denominator.

15. Solve the radical equation $\frac{3}{\sqrt{x+3}} = 1$. Be sure to check for extraneous solutions.

16. Without solving the radical equation $\sqrt{x+5} + 9 = 0$, how could you tell that it has no real solution?

Lesson Summary

To solve radical equations:

EXAMPLE: $\sqrt{12+x} = -5$

1. Isolate the radical.
2. Eliminate the radical by raising both sides to an exponent. (Example: square or cube both sides.)
3. Check for extraneous solutions.

Homework Problem Set

1.
   a. If $\sqrt{x} = 9$, then what is the value of x?
   b. If $x^2 = 9$, then what is the value of x?
   c. Is there a value of x such that $\sqrt{x+5} = 0$? If yes, what is the value? If no, explain why not.
   d. Is there a value of x such that $\sqrt{x} + 5 = 0$? If yes, what is the value? If no, explain why not.

2.
   a. Is the statement $\sqrt{x^2} = x$ true for all x-values? Explain.
   b. Is the statement $\sqrt[3]{x^3} = x$ true for all x-values? Explain.

3. Rationalize the denominator in each expression.

A. $\frac{4-x}{2+\sqrt{x}}$ B. $\frac{2}{\sqrt{x-12}}$ C. $\frac{1}{\sqrt{x+3}-\sqrt{x}}$

4. Solve each equation, and check the solutions.

A. $\sqrt{x+6}=3$

B. $2\sqrt{x+3}=6$

C. $\sqrt{x+3}+6=3$

D. $\sqrt{x+3}-6=3$

E. $16=8+\sqrt{x}$

F. $\sqrt{3x-5}=7$

G. $\sqrt{2x-3}=\sqrt{10-x}$

H. $3\sqrt{x+2}+\sqrt{x-4}=0$

I. $\frac{\sqrt{x+9}}{4}=3$

J. $\frac{12}{\sqrt{x+9}}=3$

K. $\sqrt{x^2+9}=5$

L. $\sqrt{x^2-6x}=4$

M. $\frac{5}{\sqrt{x-2}}=5$

N. $\frac{5}{\sqrt{x}-2}=5$

O. $\sqrt[3]{5x-3}+8=6$

P. $\sqrt[3]{9-x}=6$

5. Consider the inequality $\sqrt{x^2+4x}>0$. Determine whether each x-value is a solution to the inequality.

a. $x=-10$ b. $x=-4$ c. $x=10$ d. $x=4$

6. Show that $\frac{a-b}{\sqrt{a}-\sqrt{b}} = \sqrt{a} + \sqrt{b}$ for all values of a and b such that $a > 0$ and $b > 0$ and $a \neq b$.

7. Without actually solving the equation, explain why the equation $\sqrt{x+1} + 2 = 0$ has no solution.