Lesson 23: Solving Complicated Radical Equations

In the last lesson you solved equations with radicals but there was only one variable term. In this lesson, you'll expand your equation-solving skills to include equations with radical and non-radical variable terms.

Opening Exercise

1. Carlos and Andrea were solving the equation $x^2 + 2x = 0$. Andrea says that there are two solutions, 0 and -2. Carlos says the only solution is -2 because he divided both sides by *x* and got *x* + 2 = 0. Who is correct and why?

2. Carlos and Andrea are solving a different equation $\sqrt{x} = -3$. Andrea says the solution is 9 because she squared both sides and got x = 9. Carlos says there is no solution. Who is correct? Why?

3. Carlos and Andrea tried one more problem, $6 = x + \sqrt{x}$. They both agreed that you need to square both sides, but Andrea wants to isolate the $\sqrt{x}$ first and Carlos thinks they should isolate the *x*. Finish both methods and then decide which you like best. Do they both give the same answer?

Carlos' Method	Andrea's Method
$6 = x + \sqrt{x}$ $6 - \sqrt{x} = x$ $\left(6 - \sqrt{x}\right)^2 = x^2$	$6 = x + \sqrt{x}$ $6 - x = \sqrt{x}$ $(6 - x)^2 = \left(\sqrt{x}\right)^2$

4. Solve each radical equation and then write the steps needed to solve radical equations.

A. $3x = 1 + 2\sqrt{x}$

Steps to Solve Radical Equations

1. _____

_____

2. _____

_____

3. _____

_____

4. _____

_____

5. _____

_____

6. _____

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B. $3 = 4\sqrt{x} - x$

C. $\sqrt{x+5} = x - 1$

D. $\sqrt{3x+7} + 2\sqrt{x-8} = 0$

What if there is no way to isolate the radical? What is going to be the easiest way to square both sides?

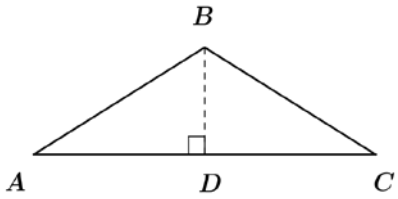
5. Solve the equation $\sqrt{x} + \sqrt{x+3} = 3$.

6. Solve the following equations.

   A. $\sqrt{x-3} + \sqrt{x+5} = 4$

   B. $3 + \sqrt{x} = \sqrt{x+81}$

7. Consider the triangle ABC shown to the right where $AD = DC$, and $\overline{BD}$ is the altitude of the triangle.



A. If the length of $\overline{BD}$ is x cm, and the length of $\overline{AC}$ is 18 cm, how long is $\overline{AD}$ and $\overline{DC}$?

B. Write an expression for the lengths of $\overline{AB}$ and $\overline{BC}$ in terms of x. (Hint: Use the Pythagorean Theorem.)

C. Write an expression for the perimeter of $\Delta\ ABC$ in terms of x.

D. Find the value of x for which the perimeter of $\Delta\ ABC$ is equal to 38 cm.

Lesson Summary

If $a = b$ and n is an integer, then $a^n = b^n$.

However, the converse is not necessarily true. The statement $a^n = b^n$ does not imply that $a = b$. Therefore, it is necessary to check for extraneous solutions when both sides of an equation are raised to an exponent.

Example: $(-3)^2 = 3^2$ but $-3 \neq 3$.

Homework Problem Set

1. Solve.

A. $\sqrt{2x-5}-\sqrt{x+6}=0$

B. $\sqrt{2x-5}+\sqrt{x+6}=0$

C. $\sqrt{x-5}-\sqrt{x+6}=2$

D. $\sqrt{2x-5}-\sqrt{x+6}=2$

E. $\sqrt{x+4}=3-\sqrt{x}$

F. $\sqrt{x+4}=3+\sqrt{x}$

G. $\sqrt{x+3} = \sqrt{5x+6} - 3$

H. $\sqrt{2x+1} = x - 1$

I. $\sqrt{x+12} + \sqrt{x} = 6$

J. $2\sqrt{x} = 1 - \sqrt{4x-1}$

K. $2x = \sqrt{4x-1}$

L. $\sqrt{4x-1} = 2 - 2x$

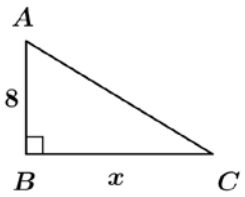
M. $x + 2 = 4\sqrt{x - 2}$

N. $\sqrt{2x - 8} + \sqrt{3x - 12} = 0$

O. $x = 2\sqrt{x - 4} + 4$

P. $x - 2 = \sqrt{9x - 36}$

2. Consider the right triangle ABC shown to the right, with $AB = 8$ and $BC = x$.



A. Write an expression for the length of the hypotenuse in terms of x.

B. Find the value of x for which $AC - AB = 9$.