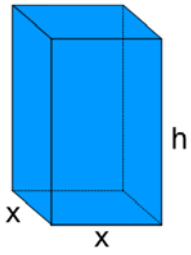
Homework Problem Set Sample Solutions

Problems 1 and 2 requires the use of a graphing calculator. If students do not have the means to complete this, the last two parts of each could be done in class.

1. **A box is to be constructed so that it has a square base and no top.**

   A. Draw and label the sides of the box. Label the sides of the base as x and the height of the box as h.



Geometry Review: The surface area is the area of all the outside faces or surfaces of the solid. In this case, the surface area is the area of all five faces. (Remember there is no top to this box.)

   B. The surface area is $108\ \text{cm}^2$. Write a formula for the surface area S, and then solve for h.

$$S = x^2 + 4xh = 108$$

$$h = \frac{108 - x^2}{4x}$$

   C. Write a formula for the function of the volume of the box in terms of x.

$$V(x) = x^2h = x^2\left(\frac{108 - x^2}{4x}\right) = \frac{108x^2 - x^4}{4x} = \frac{108x - x^3}{4}$$

   D. Use a graphing utility to find the maximum volume of the box.

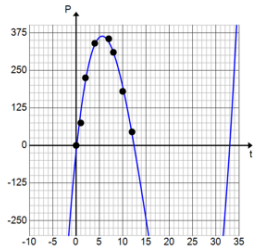
$$108\ \text{cm}^3$$

   E. What dimensions should the box be in order to maximize its volume?

$$6\ \text{cm} \times 6\ \text{cm} \times 3\ \text{cm}$$

2. The owners of Dizzy Lizzy's, an amusement park, are studying the wait time at their most popular roller coaster. The table below shows the number of people standing in line for the roller coaster t hours after Dizzy Lizzy's opens. Jaylon made a scatterplot and decided that a cubic function should be used to model the data. His scatterplot and curve are shown below.

t (hours after park opens)	P (number of people in line)
0	0
1	75
2	225
4	345
7	355
8	310
10	180
12	45



A. Do you agree that a cubic polynomial function is a good model for this data? Explain.

Yes, the curve passes through most of the points and seems to fit the data.

B. What information would Dizzy Lizzy's be interested in learning about from this graph? How could they determine the answer?

The company should be interested in the time when the line is the longest and how many people are in line at that time. To find this out, they can find a model that could be used to predict the number of people in line at any time during the day. They could then estimate the maximum point from the graph.

Scaffolding:

Have early finishers find a quadratic model for comparison. Which model seems to be a better fit? Why? Answers will vary, but one model would be $P(t) = -10.134t(t - 12)$. Students should see that the quadratic model does not seem to fit the data as well as the cubic model. They could confirm this using the regression feature of a graphing calculator.

C. Estimate the time at which the line is the longest. Explain how you know.

From the graph, the line is longest at 5.5 hours because the relative maximum of the function occurs at 5.5 hours.

D. Estimate the number of people in line at that time. Explain how you know.

From the graph, there are roughly 372 people in line when $t = 5.5$; that is the approximate relative maximum value of P.

E. What would be a reasonable domain for this function f? Why?

MP.3

A reasonable domain for f would be $0 \leq x \leq 12.5$ because the opening of the park corresponds to $t = 0$, and after 12.5 hours the park closes, so there are no people waiting in line.

F. Use the regression feature of a graphing calculator to find a cubic function f to model the data.

The calculator gives $f(t) = 0.43t^3 - 17.78t^2 + 156.63t - 24.16$.

G. Use your function f to calculate the number of people in line 10 hours after the park opens. How close is this to the actual number of a people in the table?

MP.2

The regression equation gives $f(10) = 194$ people in line 10 hours after the park opened. This is 14 people more than in the data table.

REVIEW

3. Determine the number of terms in each polynomial. If you've forgotten what polynomials are, watch the YouTube video *Algebra Basics: What are Polynomials?* By Math Antics at https://www.youtube.com/watch?v=ffLLmV4mZwU.

A. $3x^3 - 7x^2 + 2x$ 3 **B.** $6x^5 - 12x^4 + 3x^2 - 42$ 4

4. Determine the degree of each polynomial.

A. $3x^3 - 7x^2 + 2x$ 3 **B.** $6x^5 - 12x^4 + 3x^2 - 42$ 5

5. Simplify each expression by adding or subtracting like terms. Math Antics also created a video on *Simplifying Polynomials* at https://www.youtube.com/watch?v=DKC74YKJpNY.

A. $(5x^3 + 4x^2 - 7) + (12x^3 - 8x^2 + 2x + 13) =$
$17x^3 - 4x^2 + 2x + 6$

B. $(12x^2 + 8x^7 - 24x^3) + 2(4x^3 - 7x^7 + 33) =$
$-6x^7 - 16x^3 + 12x^2 + 66$

C. $(11x^5 - 20x^4 + 17x - 9) - (13x^5 - 10x^4 - 3x + 12) =$
$-2x^5 - 10x^4 + 20x - 21$

D. $3(7x^4 - 4x + 5) - (23x^2 - 8x + 50) =$
$21x^4 - 23x^2 - 4x - 35$

6. Determine the missing value that will make this equation true. Then explain how you found the correct value. $\underline{3}(3x^2 + 4x - 6) + 2(4x^2 - 5x + 2) = 17x^2 + 2x - 14$

7. Determine which functions are linear, which are quadratic and which are neither.

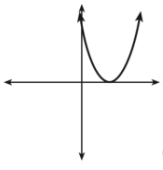
A. $x^2 + y^2 = 9$
Neither

B. $3y = 2x^2 + 6$
Quadratic

C. $3x + 2y = 6$
Linear

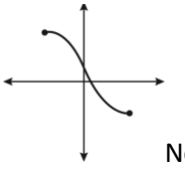
D. $y = |x - 7|$
Neither

E.



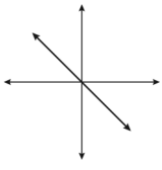
Quadratic

F.



Neither

G.



Linear

8. Complete the table of values below so that the graph of the points is a parabola.

x	3	2	1	0	-1	-2	-3
y	-8	-3	0	1	0	-3	-8