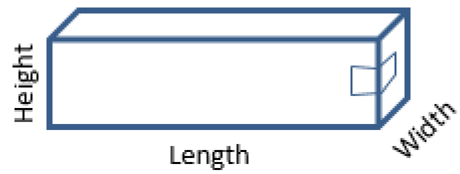
Lesson 3: Multiplying Polynomials

Opening Exercise

In Lesson 2, we found two equations for the volume of the open-topped box.



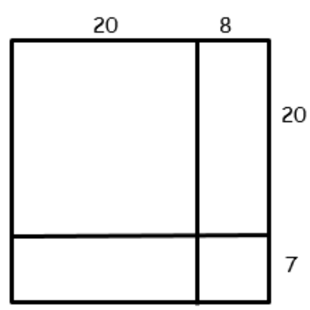
Equation from the Graphing Utility	**Equation from the Patterns in the Table**
$V = 4h^3 - 76h^2 + 325h$	$V = (25 - 2h)(13 - 2h)(h)$

V represents the volume of the open-topped box and h represents the height of the box

One way we can tell if these equations are the same is by multiplying the factors in the second equation, combining like terms and checking to see if it matches the first equation. Before we begin multiplying those three factors, we'll start with a less complicated case.

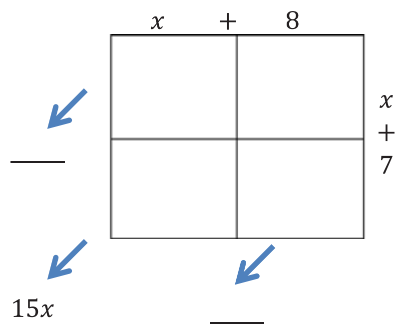
1. A. Show that $28 \times 27 = (20 + 8)(20 + 7)$ using an area model.

   B. What do the numbers you placed inside the four rectangular regions represent?



$28 \times 27 = (20 + 8)(20 + 7) =$ ______ $+$ ______ $+$ ______ $+$ ______ $=$ ______

2. A. Use the tabular method to multiply $(x + 8)(x + 7)$ and combine like terms.



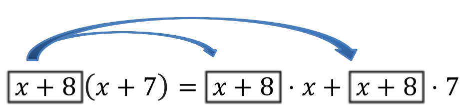
$(x+8)(x+7) =$ ______ + ______ + ______ + ______ = ______ + ______ + ______

B. Explain how the result of Part A is related to the answer of 756 in Question 1.

C. How can we multiply these binomials without using a table?

3. Jill says that we should "*Think of* $x + 8$ *as a single number and distribute over* $x + 7$." She drew the picture below to illustrate her idea.

A. What would be the next step in Jill's method?



B. Check your answer using Jill's method with the answer you got in Question 2A.

4. A. Use the tabular method to multiply $(x^2 + 3x + 1)(x^2 - 5x + 2)$ and combine like terms.

x^2	$3x$	1	
			x^2
			$-5x$
			2

$(x^2 + 3x + 1)(x^2 - 5x + 2) =$ ______ + ______ + ______ + ______ + ______

B. Why might the tabular method be easier to use than Jill's distributive method for this particular problem?

5. Use the tabular method to multiply $(x^2 + 3x + 1)(x^2 - 2)$ and combine like terms.

It will be easier to combine like terms, if you change $x^2 - 2$ to $x^2 + 0x - 2$.

$(x^2 + 3x + 1)(x^2 - 2) =$ ______ + ______ + ______ + ______ + ______

6. Use the tabular method to multiply $(a+b)(c+d)$.

$$(a+b)(c+d) = \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_$$

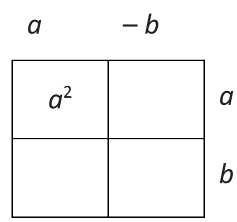
7. Let's go back to our opening question and see if the two equations from Lesson 2 are the same.

   A. Multiply $(25 - 2h)(13 - 2h)(h)$ using either method.

   B. Are the two equations the same?

Looking for Patterns

8. There are patterns that we frequently see throughout the study of Algebra 2. Use the tabular or distributive methods to find each rule. The first one has been partially done for you.

	General Rule	Model
A.	$(a-b)(a+b)=$ ______ $-$ ______	
B.	$(a+b)^2=(a+b)(a+b)=$ ______ $+$ ______ $+$ ______	
C.	$(a-b)^2=(a-b)(a-b)=$ ______ $-$ ______ $+$ ______	
D.	$(a+b)\left(a^2-ab+b^2\right)=$ ______ $+$ ______	
E.	$(a-b)\left(a^2+ab+b^2\right)=$ ______ $-$ ______	

Lesson Summary

You can use the tabular method or the distribution method to multiply polynomials.

There are several rules we discovered in this lesson. These rules will make factoring in later lessons easier.

- The Difference of Squares: $(a-b)(a+b) =$ ______ $-$ ______
- Squaring a Binomial: $(a+b)^2 = (a+b)(a+b) =$ ______ $+$ ______ $+$ ______

  $(a-b)^2 = (a-b)(a-b) =$ ______ $-$ ______ $+$ ______
- The Sum of Cubes: $(a+b)\left(a^2 - ab + b^2\right) =$ ______ $+$ ______
- The Difference of Cubes: $(a-b)\left(a^2 + ab + b^2\right) =$ ______ $-$ ______

Homework Problem Set

1. Complete the following statements by filling in the blanks.

   A. $(a+b)(c+d+e) = ac + ad + ae + \_\_\_ + \_\_\_ + \_\_\_$

   B. $(r-s)^2 = (\_\_\_\_)^2 - (\_\_\_\_)rs + s^2$

   C. $(2x+3y)^2 = (2x)^2 + 2(2x)(3y) + (\_\_\_\_)^2$

   D. $(w-1)(1+w+w^2) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_ - 1$

   E. $a^2 - 16 = (a + \_\_\_\_)(a - \_\_\_\_)$

   F. $(2x+5y)(2x-5y) = \_\_\_\_ - \_\_\_\_$

   G. $(2^{21}-1)(2^{21}+1) = \_\_\_\_ - 1$

   H. $[(x-y)-3][(x-y)+3] = (\_\_\_\_\_\_\_\_)^2 - 9$

Think about the rules above and what you reviewed in Lesson 1 about exponents.

2. Use the tabular method to multiply and combine like terms.

   A. $(x^2-4x+4)(x+3)$

   B. $(11-15x-7x^2)(25-16x^2)$

C. $(3m^3 + m^2 - 2m - 5)(m^2 - 5m - 6)$

D. $(x^2 - 3x + 9)(x^2 + 3x + 9)$

3. Multiply and combine like terms to write as the sum or difference of monomials.

A. $2a(5 + 4a)$

B. $x^3(x + 6) + 9$

C. $\frac{1}{8}(96z + 24z^2)$

D. $2^{23}(2^{84} - 2^{81})$

E. $(x - 4)(x + 5)$

F. $(10w - 1)(10w + 1)$

G. $(3z^2 - 8)(3z^2 + 8)$

H. $(-5w - 3)w^2$

I. $(m^3 - 2m + 1)(m^2 - m + 2)$

J. $(2r + 1)(2r^2 + 1)$

K. $(t - 1)(t + 1)(t^2 + 1)$

L. $n(n + 1)(n + 2)$

M. $(x + 2)(x + 2)(x + 2)$

N. $n(n + 1)(n + 2)(n + 3)$

4. Polynomial expressions can be thought of as a generalization of place value.

   A. Multiply 214×112 using the standard paper-and-pencil algorithm.

   B. Multiply $(2x^2 + x + 4)(x^2 + x + 2)$ using the tabular method and combine like terms.

   C. Substitute $x = 10$ into your answer from Part B.

   D. Is the answer to Part C equal to the answer from Part A? Compare the digits you computed in the algorithm to the coefficients of the entries you computed in the table. How do the place-value units of the digits compare to the powers of the variables in the entries?

Challenge Problem

5. In the diagram, the side of the larger square is x units, and the side of the smaller square is y units. The area of the shaded region is $(x^2 - y^2)$ square units. Show how the shaded area might be cut and rearranged to illustrate that the area is $(x - y)(x + y)$ square units.