Lesson 5: Comparing Methods—Long Division, Again?

Opening Exercise

1. Use the reverse tabular method to determine the quotient $\frac{x^3+5x^2+7x+3}{x+3}$.

$(x+3)(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_) = x^3+5x^2+7x+3$

$\left(x^3+5x^2+7x+3\right) \div (x+3) = (\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)$

Long Division

2. If $x = 10$, then the division $1573 \div 13$ can be represented using polynomial division.

If $x = 10$,

then $x + 3 =$ _______

and

$x^3+5x^2+7x+3 =$ ______________

$13\overline{)1573}$

$x+3\overline{)x^3+5x^2+7x+3}$

The long division algorithm you learned in elementary school is a special case of polynomial long division.

3. Think back to your work in Exercise 2.
   A. What expression multiplied by x will result in x^3?

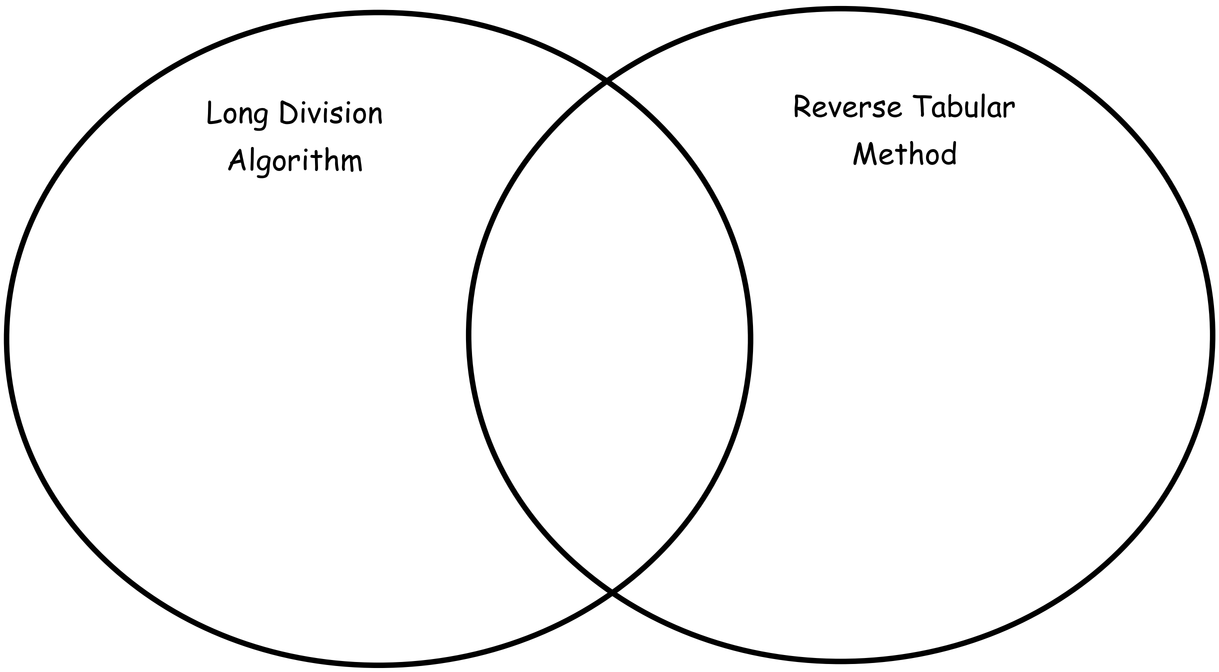
   B. When you do long division, you multiply the first digit of the quotient by the divisor and then subtract the result. It works the same with polynomial division. How do we represent multiplication and subtraction of polynomials?

   C. Then, we repeat the process to determine the next term in the quotient. What do we need to bring down to complete the process?

4. Use the long division algorithm for polynomials to evaluate

$$\frac{2x^3 - 4x^2 + 2}{2x - 2}.$$

5. Discuss with your partner the differences and similarities between the long division algorithm you used in Exercise 2 and reverse tabular method you used in Exercise 1. Fill in the Venn diagram below with your ideas – similarities go in the overlapping section, differences go in the outer sections.



For the problems below, use the reserve tabular method for one and the long division algorithm for the next. Your partner should do the reverse. Then check that you are both getting the same quotient.

6. Method Used: ___________________________

$$\frac{x^2 + 6x + 9}{x + 3}$$

7. Method Used: ___________________________

$$\frac{7x^3 - 8x^2 - 13x + 2}{7x - 1}$$

8. Method Used: ________________________

$$\frac{x^3 - 27}{x - 3}$$

9. Method Used: ________________________

$$\frac{2x^4 + 14x^3 + x^2 - 21x - 6}{2x^2 - 3}$$

CHALLENGE EXERCISES 10 – 13

10. Method Used: ________________________

$$\frac{5x^4 - 6x^2 + 1}{x^2 - 1}$$

11. Method Used: ________________________

$$\frac{x^6 + 4x^4 - 4x - 1}{x^3 - 1}$$

12. Method Used: ________________________

$$\frac{2x^7 + x^5 - 4x^3 + 14x^2 - 2x + 7}{2x^2 + 1}$$

13. Method Used: ________________________

$$\frac{x^6 - 64}{x + 2}$$

Lesson Summary

In this Lesson we learned another method for dividing polynomials – the long division algorithm. The first step has been started for you. Complete this problem and write down all the steps you used.

$$x+5\overline{)x^2+6x+5}$$

Divide x^2 by x or ask yourself "x multiplied by _____ will give me x^2?"

Homework Problem Set

Use the long division algorithm to determine the quotient in Problems 1–4.

1. $$\frac{2x^3 - 13x^2 - x + 3}{2x + 1}$$

2. $$\frac{3x^3 + 4x^2 + 7x + 22}{x + 2}$$

3. $$\frac{x^4 + 6x^3 - 7x^2 - 24x + 12}{x^2 - 4}$$

4. $$(12x^4 + 2x^3 + x - 3) \div (2x^2 + 1)$$

5. Use long division to find the polynomial, p, that satisfies the equation below. Hint: Solve for $p(x)$.

$$2x^4 - 3x^2 - 2 = (2x^2 + 1)(p(x))$$

6. Given $q(x) = 3x^3 - 4x^2 + 5x + k$.

   A. Determine the value of k so that $3x - 7$ is a factor of the polynomial q.

   B. What is the quotient when you divide the polynomial q by $3x - 7$?

Use the long division algorithm to determine the quotient in Problems 7–10.

7. $\dfrac{x^2 - 9}{x + 3}$

8. $\dfrac{x^4 - 81}{x + 3}$

9. What is similar about Problems 7 and 8?

10. $\dfrac{x^3 + 27}{x + 3}$

11. $\dfrac{x^5 + 243}{x + 3}$

12. What is similar about Problems 10 and 11?

13. Look back on your answers in Problems 7 – 10 to help you answer the following questions.

A. Is $x + 3$ a factor of $x^3 - 27$? Explain your answer using the long division algorithm.

B. Is $x + 3$ a factor of $x^2 + 9$? Explain your answer using the long division algorithm.

C. For which positive integers n is $x + 3$ a factor of $x^n + 3^n$? Explain your reasoning.

D. If n is a positive integer, is $x + 3$ a factor of $x^n - 3^n$? Explain your reasoning.