Lesson 7: Graphs of Polynomial Functions

Introduction to the Lesson

At the beginning of this unit, we looked at the graph of the volume of the open-topped box. We found two cubic equations to model that situation. The two equations – one obtained through algebra and one through a graphing utility – were then compared. Along the way, we've reviewed polynomial multiplication and division. In this lesson and the next one, we'll see how these skills will help us graph polynomial functions. We'll also learn to derive our own equations given the graph or characteristics of the function. This lesson has us look at some simple graphs and equations to see patterns and shortcuts we can use throughout Algebra 2.

Exploratory Exercise – Activity 1: Group Graph Sort

Your group will need: Graphs and Equations Handouts, scissors, tape or glue, markers, and a half sheet of poster paper

1. A. Cut out all 16 of the polynomial graphs on the Graphs Handout.

   B. Sort the graphs into 3 to 8 groups. There are only quadratics and cubic functions, so you'll have to look at other characteristics of these functions to break these two groups into subgroups. Be sure you have a reason for how you sorted them.

   C. Divide your poster paper into the appropriate number of a sections and tape the graphs in their section. Leave room to write in your reasoning.

   D. Write a category name/description for each group of graphs. This can be bullet points or a complete sentence.

2. A. Cut out the first 8 equations on the Equations Handout.

   B. Determine which equation goes with each graph. Then tape the equation to the bottom of the graph. Be sure you can still see the graph.

3. Write at least two patterns you see between the graphs and their corresponding equations.

   Pattern 1: ____

   Pattern 2: ____

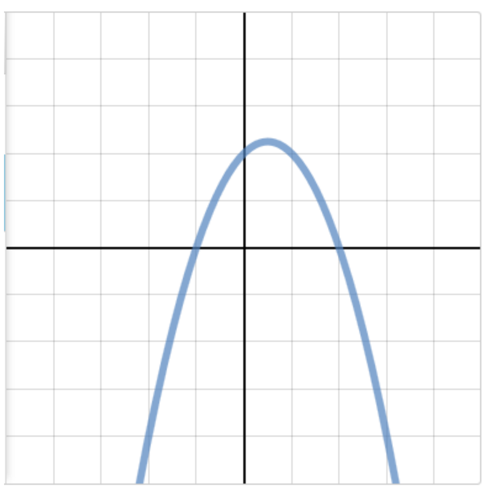
End Behavior of a Polynomial Function

- **End Behavior of a polynomial function, *f*(*x*), is defined as the actions the graph exhibits as *x* approaches infinity or negative infinity.**
- **We are looking to see if the function goes in an upward direction forever or the function goes in a downward direction forever on each side.**
- **Generally, we'll state it as "the right side is up" (or down) and the "the left side is up" (or down).**

4. What patterns can you state about parabolas in regards to end behavior?

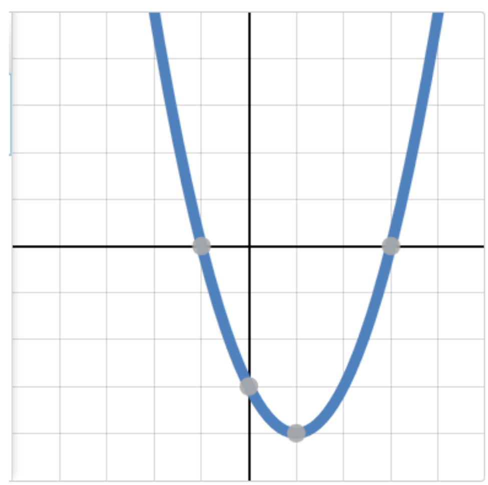
5. For the graphs below, state the end behavior.

A.



End Behavior: ___________________________________

B.



End Behavior: ___________________________________

Zeros (x-intercepts) of a Polynomial Function

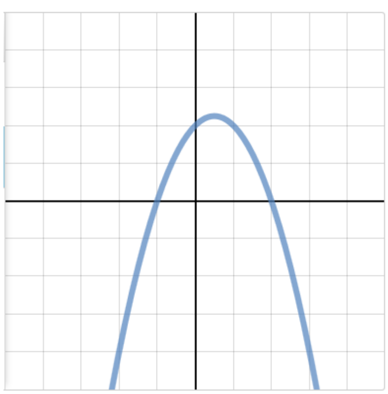
- **A zero of a polynomial function, $f(x)$, is defined as the solution to the equation $f(x) = 0$.**
- **We are looking to see where the function touches or passes through the x-axis.**
- **Generally, we'll state it as "zeros of $f(x)$", but these could be called the roots or the solutions or the x-intercepts.**
- **It is easier to find the zeros when the function is in factored form.**

6. A. Use your group's chart to mark all the zeros on the parabola graphs.

   B. How are these zeros related to the equations? Be specific and clear in your description.

7. For the graphs below, state the zeros. Then write an equation that would fit each parabola.

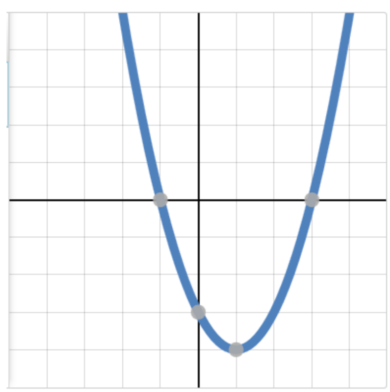
A.



Zeros: ______________________________________

Equation: ___________________________________

B.



Zeros: ______________________________________

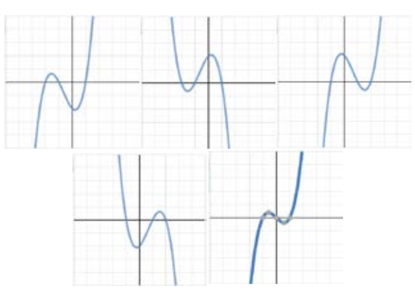
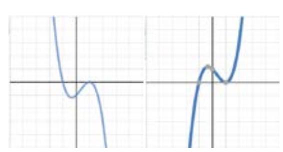
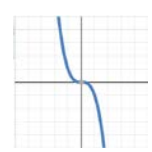
Equation: ___________________________________

8. A. Match the remaining 8 equations to the cubic functions. Be sure to check the end behavior of each one. This should follow the same pattern you wrote in Exercise 4.

   B. Cubic are 3rd degree functions while quadratics are 2nd degree functions. How does the graph show these change in the degree?

   C. Mark the zeros on the cubic graphs. Does this correlate to the equation? Does it make sense in light of your work on Exercise 6B?

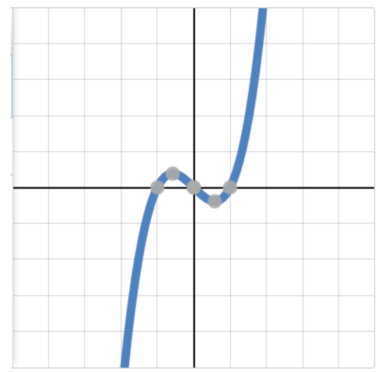
9. Gabriel sorted the cubic functions in the following way. What category system is Gabriel using? Write the categories next to the group number below.

Group 1: ____________________	Group 2: ____________________	Group 3: ____________________
		

Multiplicities of the Zeros of a Polynomial Function

- **The multiplicity of a polynomial function tells the number of times a particular number is a zero of the function.**
- **If the function touches the *x*-axis, then the multiplicity is an even number. If the function passes through the *x*-axis, then the multiplicity is an odd number.**

Example: In Graph F, as shown at the right, there are zeros at -1, 0 and 1. The function goes through the *x*-axis at each zero so the multiplicity of each one is an odd number. Since we know the equation for this function is $y = x(x-1)(x+1)$, we know that the multiplicity of each zero is 1, since there is only 1 of each of them.



10. In Exercise 6 and 8C, your team marked all the zeros on the functions. On your poster, state the multiplicity of each zero based on the equation.

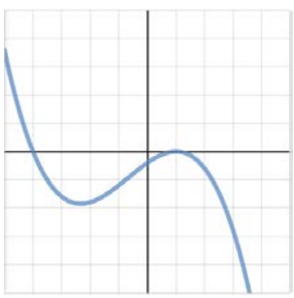
Lesson Summary

11. Fill in the chart below with sketches of each situation. There are two where no graph is possible.

End Behavior	All Even Multiplicities	All Odd Multiplicities	At Least One Even & At Least One Odd Multiplicity
Both Up			
Both Down			
Right Up & Left Down			
Right Down & Left Up			

Homework Problem Set

For each problem below, extend what you learned in this lesson to determine the zeros and end behavior.

	Graph or Equation	Zeros	End Behavior
1.	$y = -x(x+3)(x-2)(x+1)$		
2.			
3.	$y = x^2(x-1)$		
4.			
5.	$y = (x-1)(x+1)(x-2)(x+3)$		

6. A. Write an equation that has zeros at 2, -3 and 0 and the right side is going up.

B. Is it possible to have another equation with the same zeros and the right side going down? Explain your thinking.

Spiral Review

7. Use exponent rules to simplify each expression. (Lesson 1 Homework Problem Set)

A. $3x^4 \bullet x^0 \bullet x^2$

B. $y^{-5} \bullet y^{12}$

C. $\left(mn^2\right)^{-4}$

D. $1000x^0$

E. $x^{-2} \bullet y^3$

F. $\left(\dfrac{n^{\;2}}{m^{-3}}\right)^{-4}$

8. Simplify each expression. (Lesson 2 Homework Problem Set)

A. $-2\left(3x^3+4x-7\right)+x\left(2x^2-3x+1\right)$

B. $\frac{1}{2}\left(4x^2-6x+8\right)-3\left(5x^2+7\right)$

C. $2x^3-8x^2-14x+8-\left(x^3-4x\right)-\left(2x\right)$

D. $3x^3+4x^4-\left(3x^4+4x^3\right)$