Lesson 8: The Special Role of Zero in Graphing

Classwork

Opening Exercise

We have seen that having a polynomial function in factored form can help us find the zeros easily. In the next lesson, we'll use this skill to graph polynomial functions by hand. For now, we'll review what we learned in Algebra 1 about factoring.

1. Find all solutions to the equation $(x^2 + 5x + 6)(x^2 - 3x - 4) = 0$.

$(x^2+5x+6) \cdot (x^2-3x-4) =$

product is 6 & sum is 5 | product is -4 & sum is -3

$(x+\_\_\_\_\_)(x+\_\_\_\_\_)(x+\_\_\_\_\_)(x-\_\_\_\_\_)$

$x =$ _____

2. Find the solutions of $(x^2 - 9)(x^2 - 16) = 0$.

From Lesson 3, we reviewed the difference of square rule: $(a + b)(a - b) = a^2 - b^2$.

Discussion

3. Suppose we know that the polynomial equation $4x^3 - 12x^2 + 3x + 5 = 0$ has three real solutions and that one of the factors of $4x^3 - 12x^2 + 3x + 5$ is $(x - 1)$. How can we find all three solutions to the given equation?

4. A. Find all three solutions to the given polynomial equation $4x^3 - 12x^2 + 3x + 5 = 0$ if you know that $(x - 1)$ is one of the factor. (If you need a reminder, turn to Lessons 4 or 5.)

$$4x^3 - 12x^2 + 3x + 5 = (x - 1)(\_\_\_\_\_)(\_\_\_\_\_)$$

B. What are the zeros of the polynomial function. $f(x) = 4x^3 - 12x^2 + 3x + 5$? _____

5. Joey ran into some trouble when he tried to find the solutions to the equation

$$\boldsymbol{(x^2 - 6x + 3)(2x^2 - 4x - 7) = 0}.$$

A. What about this equation was difficult for Joey?

B. Write the formula Joey could use to find the solutions of this equation.

C. Use the formula to find all the solutions of this equation.

Factoring by Grouping

6. Find all solutions to $x^3 + 3x^2 - 9x - 27 = 0$ by factoring the equation. Group the first two terms together and the last two terms together and then look for a common factor in each group.

$$x^3 + 3x^2 - 9x - 27 =$$

$$\left(x^3 + 3x^2\right) - \left(9x + 27\right) =$$

$$\_\_\_\left(x+3\right) - \_\_\_\left(x+3\right) =$$

x = _____

7. Find all real solutions to $x^3 - 5x^2 - 4x + 20 = 0$.

8. Find all real solutions to $x^3 - 8x^2 - 2x + 16 = 0$.

Lesson Summary

There are many formulas you can use when factoring. Below are a few that extremely helpful in Algebra 2.

Quadratic Formula

When $ax^2+bx+c=0$ then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ where a, b and c are real numbers.

Patterns in Factoring

- The Difference of Squares: $(a-b)(a+b)=a^2-b^2$
- Squaring a Binomial: $(a+b)^2=(a+b)(a+b)=a^2+2ab+b^2$

  $(a-b)^2=(a-b)(a-b)=a^2-2ab+b^2$
- The Sum of Cubes: $(a+b)\left(a^2-ab+b^2\right)=a^3+b^3$
- The Difference of Cubes: $(a-b)\left(a^2+ab+b^2\right)=a^3-b^3$

Homework Problem Set

For Problems 1–4, find all solutions to the given equations.

1. $(x-3)(x+2)=0$

2. $(x-5)(x+2)(x+3)=0$

3. $(2x-4)(x+5)=0$

4. $(2x-2)(3x+1)(x-1)=0$

5. Find all real solutions to the equation $(x^2-9)(x^4-16)=0$.

6. Find the zeros with multiplicity for the function $p(x)=(x^3-8)(x^5-4x^3)$.

7. Find two different polynomial functions that have zeros at 1, 3, and 5 of multiplicity 1.

8. Find two different polynomial functions that have a zero at 2 of multiplicity 5 and a zero at -4 of multiplicity 3.

9. Find all real solutions to the equation $(x^2 - 9)(x^3 - 8) = 0$.

10. Solve each of the following equations using the quadratic formula.

    a. $x^2 - 5x - 3 = 0$

    b. $(6x^2 - 7x + 2)(x^2 - 5x + 5) = 0$

    c. $(3x^2 - 13x + 14)(x^2 - 4x + 1) = 0$

11. Solve the following equations by bringing all terms to one side of the equation and factoring out the greatest common factor.

    a. $(x-2)(x-1)=(x-2)(x+1)$

    b. $(2x+3)(x-4)=(2x+3)(x+5)$

    c. $(x-1)(2x+3)=(x-1)(x+2)$

    d. $(x^2+1)(3x-7)=(x^2+1)(3x+2)$

    e. $(x+3)(2x^2+7)=(x+3)(x^2+8)$

12. All of the expressions in the equations below can be factored using the techniques discussed so far in this course. Factor the expression, and find all real solutions to the equation.

A. $x^2 - 5x - 24 = 0$	B. $3x^2 + 5x - 2 = 0$
C. $x^3 + 3x^2 + 2x + 6 = 0$	D. $2x^3 + x^2 - 6x - 3 = 0$
E. $8x^3 - 12x^2 + 2x - 3 = 0$	F. $6x^3 + 8x^2 + 15x + 20 = 0$

CHALLENGE PROBLEMS

13. If p, q, r, s are nonzero numbers, find the solutions to the equation $(px + q)(rx + s) = 0$ in terms of p, q, r, s.

Use the identity $a^2 - b^2 = (a - b)(a + b)$ to solve the equations given below.

14. $(3x - 2)^2 = (5x + 1)^2$

15. $(x + 7)^2 = (2x + 4)^2$