Exit Ticket Sample Solutions

Suppose that a polynomial function p can be factored into seven factors: $(x-3)$, $(x+1)$, and 5 factors of $(x-2)$. What are its zeros with multiplicity, and what is the degree of the polynomial? Explain how you know.

Zeros: 3 with multiplicity 1; -1 with multiplicity 1; 2 with multiplicity 5

The polynomial has degree seven. There are seven linear factors as given above, so $p(x) = (x-3)(x+1)(x-2)^5$.

If the factors were multiplied out, the leading term would be x^7, so the degree of p is 7.

Homework Problem Set Sample Solutions

For Problems 1–4, find all solutions to the given equations.

1. $(x-3)(x+2)=0$

   $3, -2$

2. $(x-5)(x+2)(x+3)=0$

   $5, -2, -3$

3. $(2x-4)(x+5)=0$

   $2, -5$

4. $(2x-2)(3x+1)(x-1)=0$

   $1, -\frac{1}{3}, 1$

5. Find four solutions to the equation $(x^2-9)(x^4-16)=0$.

   $2, -2, 3, -3$

6. **Find the zeros with multiplicity for the function $p(x) = (x^3 - 8)(x^5 - 4x^3)$.**

We can factor p to give $p(x) = x^3(x-2)(x^2+2x+4)(x-2)(x+2)$.

Then, 0 is a zero of multiplicity 3, -2 is a zero of multiplicity 1, and 2 is a zero of multiplicity 2.

7. **Find two different polynomial functions that have zeros at 1, 3, and 5 of multiplicity 1.**

$p(x) = (x-1)(x-3)(x-5)$ and $q(x) = (x^2+1)(x-1)(x-3)(x-5)$

8. **Find two different polynomial functions that have a zero at 2 of multiplicity 5 and a zero at -4 of multiplicity 3.**

$p(x) = (x-2)^5(x+4)^3$ and $q(x) = (x^2+1)(x-2)^5(x+4)^3$

9. **Find all real solutions to the equation $(x^2-9)(x^3-8) = 0$.**

From Lesson 6, we know that $(x-2)$ is a factor of (x^3-8), so three solutions are $3, -3$, and 2.

10. **Solve each of the following equations using the quadratic formula.**

a. $x^2 - 5x - 3 = 0$

$x = \dfrac{5 \pm \sqrt{37}}{2}$

b. $(6x^2 - 7x + 2)(x^2 - 5x + 5) = 0$

$x = \dfrac{2}{3}, \dfrac{1}{2}, \dfrac{5 \pm \sqrt{5}}{2}$

c. $(3x^2 - 13x + 14)(x^2 - 4x + 1) = 0$

$x = 2, \dfrac{7}{3}, 2 - \sqrt{3}, 2 + \sqrt{3}$

11. **Solve the following equations by bringing all terms to one side of the equation and factoring out the greatest common factor.**

a. $(x-2)(x-1) = (x-2)(x+1)$

$$(x-2)(x+1) - (x-2)(x-1) = 0$$
$$(x-2)\big(x+1-(x-1)\big) = 0$$
$$(x-2)(2) = 0$$
$$x = 2$$

So, the only solution to $(x-2)(x-1) = (x-2)(x+1)$ is 2.

b. $(2x+3)(x-4)=(2x+3)(x+5)$

$(2x+3)(x-4)-(2x+3)(x+5)=0$

$(2x+3)\big(x-4-(x+5)\big)=0$

$(2x+3)(-9)=0$

$x=-\frac{3}{2}$

So, the only solution to $(2x+3)(x-4)=(2x+3)(x+5)$ is $-\frac{3}{2}$.

c. $(x-1)(2x+3)=(x-1)(x+2)$

$(x-1)(2x+3)-(x-1)(x+2)=0$

$(x-1)\big(2x+3-(x+2)\big)=0$

$(x-1)(x+1)=0$

$x=1$ or $x=-1$

The solutions to $(x-1)(2x+3)=(x-1)(x+2)$ are 1 and -1.

d. $(x^2+1)(3x-7)=(x^2+1)(3x+2)$

$(x^2+1)(3x-7)-(x^2+1)(3x+2)=0$

$(x^2+1)\big(3x-7-(3x+2)\big)=0$

$(x^2+1)(-9)=0$

$x^2+1=0$

There are no real number solutions to $(x^2+1)(3x-7)=(x^2+1)(3x+2)$.

e. $(x+3)(2x^2+7)=(x+3)(x^2+8)$

$(x+3)(2x^2+7)-(x+3)(x^2+8)=0$

$(x+3)\big(2x^2+7-(x^2+8)\big)=0$

$(x+3)(x^2-1)=0$

$(x+3)(x-1)(x+1)=0$

The three solutions to $(x+3)(2x^2+7)=(x+3)(x^2+8)$ are -3, -1, and 1.

12. **All of the expressions in the equations below can be factored using the techniques discussed so far in this course. Factor the expression, and find all real solutions to the equation.**

A. $x^2 - 5x - 24 = 0$ $(x-8)(x+3) = 0$. Solutions: $8,\ -3$	B. $3x^2 + 5x - 2 = 0$ $(3x-1)(x+2) = 0$. Solutions: $\frac{1}{3}, -2$
C. $x^3 + 3x^2 + 2x + 6 = 0$ $(x+3)(x^2+2) = 0$. Solution: -3	D. $2x^3 + x^2 - 6x - 3 = 0$ $(2x+1)(x-\sqrt{3})(x+\sqrt{3}) = 0$. Solutions: $-\frac{1}{2}, \sqrt{3}, -\sqrt{3}$
E. $8x^3 - 12x^2 + 2x - 3 = 0$ $(2x-3)(4x^2+1) = 0$. Solution: $\frac{3}{2}$	F. $6x^3 + 8x^2 + 15x + 20 = 0$ $(3x+4)(2x^2+5) = 0$. Solution: $-\frac{4}{3}$

CHALLENGE PROBLEMS

13. **If p, q, r, s are nonzero numbers, find the solutions to the equation $(px+q)(rx+s) = 0$ in terms of p, q, r, s.**

Setting each factor equal to zero gives solutions $-\frac{q}{p}$ and $-\frac{s}{r}$.

Use the identity $a^2 - b^2 = (a-b)(a+b)$ to solve the equations given in Problems 12–13.

14. $(3x-2)^2 = (5x+1)^2$

Using algebra, we have $(3x-2)^2 - (5x+1)^2 = 0$. Applying the difference of squares formula, we have $\big((3x-2)-(5x+1)\big)\big((3x-2)+(5x+1)\big) = 0$. Combining like terms gives $(-2x-3)(8x-1) = 0$, so the solutions are $-\frac{3}{2}$ and $\frac{1}{8}$.

15. $(x+7)^2 = (2x+4)^2$

Using algebra, we have $(x+7)^2 - (2x+4)^2 = 0$. Then $\big((x+7)-(2x+4)\big)\big((x+7)+(2x+4)\big) = 0$, so we have $(-x+3)(3x+11) = 0$. Thus the solutions are $-\frac{11}{3}$ and 3.