Algebra II/Trig Name ______________________

Geometric Sequences
Module 2, Unit 3, Lesson 4

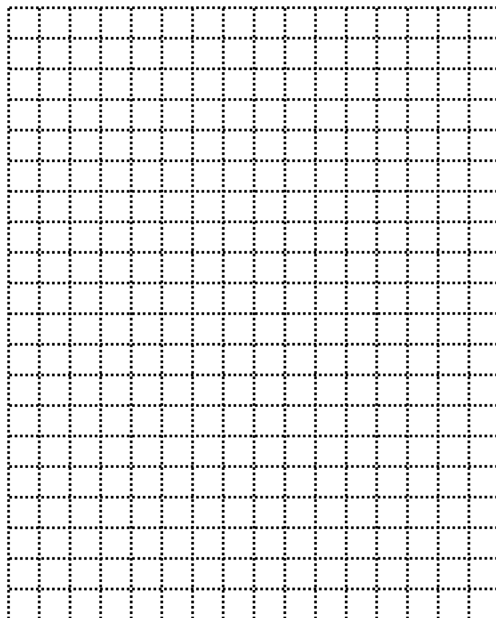
Example 1: Use this sequence to answer the following questions: 2, 6, 18, …

a. Write the next three terms.

b. What is the pattern? Is this an arithmetic sequence? Why or why not?

c. Fill in the table below. Then plot the points on the graph. Consider an appropriate scale.

n	f(n)
1	2
2	6
3	18
4	
5	
6	



d. What type of graph do the points make?

e. So now we know…

arithmetic sequences are modeled by ________________ functions

geometric sequences are modeled by ________________ functions

Example 2: Identify each of the following as arithmetic, geometric, or neither.

a. $5, -10, 20, -40, ...$

b. $1, 3, 7, 13, ...$

c. $5, -2, -9, -16, ...$

Definition of a Geometric Sequence
A geometric sequence is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed nonzero constant. The amount by which we multiply each time is called the common ratio of the sequence.

Geometric sequences can be modeled by exponential functions.

Finding the Common Ratio
The common ratio, r, is found by dividing any term after the first term by the term that directly precedes it.

$$r = \frac{a_2}{a_1}$$

Example 3: Find the common ratio for the following geometric sequences:
a. 1, 5, 25, 125, 625, ……

b. 9, -3, 1, $\frac{-1}{3}$, $\frac{1}{9}$, ……

Example 4: Write the first six terms of the geometric sequence with first term 6 and common ratio of 3.

General Term of a Geometric Sequence
The nth term (the general term) of a geometric sequence with first term a_1 and common ratio r is

$$a_n = a_1 r^{n-1}$$

Example 5: Find the a_8 of the geometric sequence when $a_1 = -4$ and whose common ratio is –2.

Example 6: Write a formula for the nth term of the sequence below. Then find a_8.

a. $12, 6, 3, \frac{3}{2}, \ldots\ldots$

b. 3, 6, 12, 24, 48, ……

The Sum of the First *n* Terms of a Geometric Sequence

The sum, S(*n*), of the first *n* terms of a geometric sequence is given by

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

in which a_1 is the first term and *r* is the common ratio ($r \neq 1$).

Example 7: Find the sum of the first 8 terms of the geometric sequence:
2, -8, 32, -128, ……..

Example 8: Find the sum of the first 9 terms of the geometric sequence:
2, -6, 18, -54, ……..

Example 9: Find the following sum:

a. $\sum_{i=3}^{10} 6 \bullet 2^i$

b. $\sum_{i=2}^{6} \left(\frac{1}{3}\right)^{i+1}$

The Sum of an Infinite Geometric Series
If -1 < r < 1, then the sum of the infinite geometric series

$$a_1 + a_1 r + a_1 r^2 + a_1 r^3 + ...$$

in which a_1 is the first term and r is the common ratio is given by

$$S = \frac{a_1}{1-r}$$

Example 10: Find the sum of the sequence: $\frac{3}{8} - \frac{3}{16} + \frac{3}{32} - \frac{3}{64} + ...$

Example 11: Find the sum of the sequence: $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + ...$

Practice – Bringing It All Together

Are the following sequences arithmetic, geometric, or neither?

1. $4, \frac{5}{2}, 1, -\frac{1}{2}, ...$
2. 8, 12, 18, 27, …
3. 2, 5, 10, 17, 26, …

4. A display of cans on a grocery shelf consists of 20 cans on the bottom, 18 can in the next row, and so on until the top row has 4 cans. How many cans, in total, are in the display?

5. Hector gets better and better at a video game every time he plays. He scores 20 points in the first game, 25 in the second, 30 in the third and so on. How many points will he score in his 15^{th} game?

6. A virus reproduces by diving into two, and after a certain growth period, it divides in two again. How many viruses will in a system starting with a single virus after 10 division?