

Precalculus Preview  
Logarithms**Logarithms**

Logarithms are the "opposite" of exponentials, just as subtraction is the opposite of addition and division is the opposite of multiplication. Logs "undo" exponentials. Technically speaking, logs are the inverses of exponentials.

$$y = \log_b x \text{ is equivalent to } x = b^y$$

**Properties of Logarithms**

- $\log_b 1 = 0$
- $\log_b b^x = x$
- $b^{\log_b x} = x$

**Example 1:** Evaluate each expression without using a calculator.

a.  $\log_2 16$

b.  $7^{\log_7 23}$

c.  $\log_3 \frac{1}{\sqrt{3}}$

d.  $\log_4 4^5$

e.  $10^{\log \sqrt{x}}$

f.  $\log 10^7$

g.  $\ln \frac{1}{e^7}$

h.  $\ln e^{13x}$

i.  $\log 1000$

j.  $e^{\ln 125}$

k.  $\log_4 64$

l.  $\log_7 \frac{1}{\sqrt[4]{49}}$

**Product Rule**  $\log_b(MN) = \log_b M + \log_b N$

**Quotient Rule**  $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

**Power Rule**  $\log_b M^p = p \log_b M$

**Example 2:** Use properties of logarithms to expand each logarithmic expression as much as possible.

a.  $\log_5(7 \cdot 3)$

b.  $\log_7\left(\frac{7}{x}\right)$

c.  $\log_b x^3$

d.  $\log_5\left(\frac{\sqrt{x}}{25}\right)$

e.  $\log_2 \sqrt[5]{\frac{xy^4}{16}}$

f.  $\log\left[\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right]$

g.  $\log_8\left(\frac{64}{\sqrt{x+1}}\right)$

h.  $\ln\left[\frac{x^3\sqrt{x^2+1}}{(x+1)^4}\right]$

**Example 2:** Use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1.

a.  $5\ln x - 2\ln y$

b.  $\frac{1}{2}(\log x + \log y)$

c.  $\log(2x+5) - \log x$

d.  $\frac{1}{3}(\log_4 x - \log_4 y) + 2\log_4(x+1)$

e.  $\log x + \log(x^2 - 1) - \log 7 - \log(x+1)$

f.  $4\ln x + 7\ln y - 3\ln z$

g.  $\frac{1}{3}(\log_4 x - \log_4 y)$

h.  $2\ln x - \frac{1}{2}\ln y$