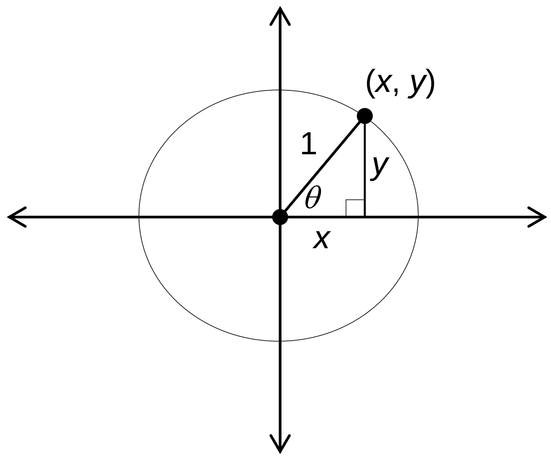
Algebra II/Trig B Name ______________________

Trigonometric Identities
Module 3, Unit 7, Lesson 11

Unit Circle



$$x^2 + y^2 = 1$$

Pythagorean Identities

Other relationships among trigonometric functions follow from the equation of the unit circle.

$$x^2 + y^2 = 1$$

Because $\cos\theta = x$ and $\sin\theta = y$, we see that

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

We will eliminate the parenthesis in this identity by writing $\cos^2\theta$ instead of $(\cos\theta)^2$ and $\sin^2\theta$ instead of $(\sin\theta)^2$.

$$\cos^2\theta + \sin^2\theta = 1$$

or

$$\sin^2\theta + \cos^2\theta = 1$$

Two additional identities can be obtained from $x^2 + y^2 = 1$.

$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\theta + \cos^2\theta = 1$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Example 1: Given that $\sin t = \frac{3}{5}$ and $0 \le t < \frac{\pi}{2}$, find the value of $\cos t$ using a trigonometric identity.

Example 2: Given that $\cos t = \frac{1}{2}$ and $0 \le t < \frac{\pi}{2}$, find the value of $\sin t$ using a trigonometric identity.

Reciprocal Identities

$\sin x = \dfrac{1}{\csc x}$	$\cos x = \dfrac{1}{\sec x}$	$\tan x = \dfrac{1}{\cot x}$
$\csc x = \dfrac{1}{\sin x}$	$\sec x = \dfrac{1}{\cos x}$	$\cot x = \dfrac{1}{\tan x}$

Quotient Identities

$\tan x = \dfrac{\sin x}{\cos x}$	$\cot x = \dfrac{\cos x}{\sin x}$

Simplifying Trigonometric Expressions

The main goal in dealing with trigonometric expressions is to simplify them. This means large, multiple-function expressions are considered simplified when they are compact and contain fewer trigonometric functions.

Strategies for Simplifying Trigonometric Expressions

1. Change the expression into sine and cosine.
2. Look to use known formulas for purposes of substitution.
3. If there are fractions, create a common denominator.
4. Use algebraic manipulations, like factoring and distributing.
5. If a strategy or substitution proves not to help, try something different.

Example 3: Simplify the expression.

a. $\cos x + \sin x \tan x$

b. $\tan x \cos x \sin x$

c. $\dfrac{\tan^2 t + 1}{1 + \cot^2 t}$

d. $\dfrac{1 - \cos^2 t}{\sin^2 t}$

e. $\sec x \tan x \cos x$

f. $\tan^2 t(\csc^2 t - 1)$

g. $\dfrac{\cos^2 x - 1}{\sin^2 x - 1}$

h. $\cos t \csc t(\sec^2 t - 1)$

i. $\dfrac{1-\tan^2 t}{1+\tan^2 t} + 1$

j. $\dfrac{\tan x}{\sec x}$

k. $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{1+\sin x}$

l. $\dfrac{\sec^2 \theta - 1}{\sec^2 \theta}$