

7.12 Proving Trigonometric Identities

Reciprocal Identities

$\sin x = \frac{1}{\csc x}$	$\cos x = \frac{1}{\sec x}$	$\tan x = \frac{1}{\cot x}$
$\csc x = \frac{1}{\sin x}$	$\sec x = \frac{1}{\cos x}$	$\cot x = \frac{1}{\tan x}$

Quotient Identities

$\tan x = \frac{\sin x}{\cos x}$	$\cot x = \frac{\cos x}{\sin x}$
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Pythagorean Identities

$\sin^2 \theta + \cos^2 \theta = 1$ $1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$

Proving Trigonometric Identities

To prove an identity, you have to use **logical** steps to show that one side of the equation in the identity can be transformed into the other side of the equation using already established identities such as the Pythagorean identity.

Guidelines for Proving Trigonometric Identities

1. Work on **one side of the equation**. Start with the more complicated side and transform it in a step-by-step fashion until it looks exactly like the other side.
2. Analyze the identity and look for opportunities to apply the fundamental identities.
3. Try using one or more of the following techniques:
 - a. Rewrite the more complicated side in terms of sines or cosines.
 - b. Factor out the greatest common factor.
 - c. Separate a single-term quotient into two terms:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{and} \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

- d. Combine fractional expressions using the least common denominator.
- e. Multiply the numerator and the denominator by a binomial factor that appears on the other side of the identity.

4. Don't be afraid to stop and start over if you are not getting anywhere.

Example 1: Verify the identity.

1. $\sec x \cot x = \csc x$

2. $\csc x \tan x = \sec x$

3. $(\csc \theta + \cot \theta)(1 - \cos \theta) = \sin \theta$

4. $\cot x \sec x \sin x = 1$

5. $\csc x - \csc x \cos^2 x = \sin x$

6. $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

7. $\tan \theta + \cot \theta = \sec \theta \csc \theta$

8. $\frac{\cos \theta \sec \theta}{\cot \theta} = \tan \theta$

$$9. \cos^2 \theta (1 + \tan^2 \theta) = 1$$

$$10. \cos t \cot t = \frac{1 - \sin^2 t}{\sin t}$$

$$11. \frac{\sec^2 t}{\tan t} = \sec t \csc t$$

$$12. \frac{\cot^2 t}{\csc t} = \csc t - \sin t$$

$$13. \frac{1 - \sin \theta}{\cos \theta} = \sec \theta - \tan \theta$$

$$14. \frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \sin t + \cos t$$