Name Key Per. ____ Date_____ Score ____

Unit 3 Worksheet

Write the first five terms of each geometric sequence.

1) $a_1 = 24, \quad r = \frac{1}{3}$

$24, 8, \frac{8}{3}, \frac{8}{9}, \frac{8}{27}$

2) $a_1 = -\frac{1}{8}, \quad r = -2$

$-\frac{1}{8}, +\frac{2}{8}, -\frac{4}{8}, +\frac{8}{8}, -\frac{16}{8}$

$-\frac{1}{8}, \frac{1}{4}, -\frac{1}{2}, 1, -2$

Use the formula for the general term of a geometric sequence to find the indicated term of each sequence with the given first term, a_1, and common ratio, r.

3) Find a_{12}, when $a_1 = 4, \ r = -2$.

$a_n = 4(-2)^{n-1}$

$a_{12} = 4(-2)^{12-1}$

$a_{12} = -8192$

4) Find a_{30}, when $a_1 = -8000, \ r = \frac{1}{2}$.

$a_n = -8000\left(\frac{1}{2}\right)^{n-1}$

$a_{30} = -8000\left(\frac{1}{2}\right)^{30-1}$

$a_{30} = -\frac{125}{8388608}$

Write a formula for the general term (the nth term) of each geometric sequence. Then use the formula for a_n to find a_9, the 9th term of the sequence.

5) $5, -1, \frac{1}{5}, -\frac{1}{25}, \ldots$

$a_n = 5\left(-\frac{1}{5}\right)^{n-1}$

$a_9 = 5\left(-\frac{1}{5}\right)^{9-1}$

$a_9 = \frac{1}{78125}$

6) $0.07, 0.007, 0.0007, 0.00007, \ldots$

$a_n = 0.07(0.1)^{n-1}$

$a_9 = 0.07(0.1)^{8}$

$a_9 = 0.0000000007$

Find the sum of the finite geometric series.

7) Find the sum of the first 12 terms of the geometric sequence:

$-3, 6, -12, 24, \ldots$

$S_{12} = \frac{-3\left(1-(-2)^{12}\right)}{1-(-2)}$

$S_{12} = \frac{-3(1-4096)}{3} = 4095$

8) Find the sum of the first 11 terms of the geometric sequence:

$-\frac{1}{24}, \frac{1}{12}, -\frac{1}{6}, \frac{1}{3}, \ldots$

$S_{11} = \frac{-\frac{1}{24}\left(1-(-2)^{11}\right)}{1-(-2)}$

$S_{11} = -\frac{1(2049)}{24(3)} =$

$S_{11} = -\frac{683}{24}$

Find the sum of the finite geometric series.

9) $\sum_{i=1}^{6} 4^i$ $a_1 = 4$ $r = 4$ $n = 6$

$S_6 = \frac{4(1-4^6)}{1-4}$

$S_6 = \frac{4(-4095)}{-3} = \boxed{5460}$

10) $\sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i+1}$ $a_1 = \frac{1}{9}$ $r = \frac{1}{3}$

$S_\infty = \frac{\frac{1}{9}}{1-\frac{1}{3}} =$

$S_\infty = \frac{1}{9} \div \frac{2}{3} = \boxed{\frac{1}{6}}$

Find the sum of the infinite geometric series.

11) $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots$

$S_\infty = \frac{1}{1-\frac{1}{4}} = 1 \div \frac{3}{4} =$

$\boxed{\frac{4}{3}}$

12) $3-1+\frac{1}{3}-\frac{1}{9}+\ldots$

$S_\infty = \frac{3}{1-(-\frac{1}{3})} = 3 \div \frac{4}{3} =$

$\boxed{\frac{9}{4}}$

Are the following sequences arithmetic, geometric, or neither? State the sum or difference.

13) $-\frac{1}{3}, -\frac{1}{5}, -\frac{1}{7}, -\frac{1}{9}, \ldots$

neither

14) $-\frac{2}{3}, -\frac{5}{3}, -\frac{8}{3}, -\frac{11}{3}, \ldots$

$\boxed{d = -1}$

$\boxed{\text{arithmetic}}$

Write a formula for the general term (the nth term) of each arithmetic sequence.

15) $a_7 = -12$, $a_{12} = 38$

$d = \frac{38+12}{12-7} = \frac{50}{5} = 10$

$38 = a_1 + (12-1)(10)$

$38 = a_1 + 110$

$a_1 = -72$

$a_n = -72 + (n-1)(10)$

$a_n = -72 + 10n - 10$

$\boxed{a_n = 10n - 82}$

16) $a_{20} = 38$, $a_{23} = -10$ $d = \frac{-10-38}{23-20} = \frac{-48}{3} = -16$

$38 = a_1 + (20-1)(-16)$

$38 = a_1 - 304$

$a_1 = 342$

$a_n = 342 + (n-1)(-16)$

$\boxed{a_n = -16n + 358}$

Find the sum.

17) $\sum_{k=1}^{10} (-3+2k)$ Arithmetic

$a_1 = -1$ $n = 10$ $a_{10} = 17$

$S_{10} = \frac{10}{2}(-1+17)$

$\boxed{S_{10} = 80}$

18) $\sum_{i=1}^{10} \frac{1}{3}(-2)^i$ Geometric

$a_1 = -\frac{2}{3}$ $r = -2$ $n = 10$

$S_{10} = \frac{-\frac{2}{3}(1-(-2)^{10})}{1-(-2)}$

$S_{10} = \frac{-2(-1023)}{3(-3)} =$

$\boxed{-\frac{682}{3}}$

Simplify. Show all necessary work. Answers should contain only positive exponents.

19) $\left(\frac{x^5}{2x^{-2}}\right)^{-3}$

$\left(\frac{x^7}{2}\right)^{-3} =$

$\left(\frac{2}{x^7}\right)^3 = \boxed{\frac{8}{x^{21}}}$

20) $4(x^2)^{-5}(2x)^{-4}$

$\frac{4(x^{-10})}{(2x)^4} =$

$\frac{4}{16x^4x^{10}} =$

$\boxed{\frac{1}{4x^{14}}}$

21) $(4x^3)(2x^5)(3x)^2$

$(4x^3)(2x^5)(9x^2)$

$\boxed{72x^{10}}$

22) $\left(\frac{3x^2y^{-4}}{2x^3y}\right)^{-2}$

$\left(\frac{2x^3y}{3x^2y^{-4}}\right)^2 =$

$\left(\frac{2xy^5}{3}\right)^2 =$

$\boxed{\frac{4x^2y^{10}}{9}}$

23) $\sqrt[3]{-48x^5y^{10}}$

$\sqrt[3]{-48} \cdot \sqrt[3]{x^5} \cdot \sqrt[3]{y^{10}}$

$-2\sqrt[3]{6} \cdot x\sqrt[3]{x^2} \cdot y^3\sqrt[3]{y}$

$\boxed{-2xy^3\sqrt[3]{6x^2y}}$

24) $\left(3^{\frac{3}{4}}\right)\left(9^{\frac{1}{4}}\right)$

$\left(3^{3/4}\right)\left(3^2\right)^{1/4} =$

$\left(3^{3/4}\right)\left(3^{2/4}\right) =$

$\boxed{3^{5/4}}$

25) $\frac{2x^{-\frac{1}{2}}}{6x^{\frac{2}{3}}} =$

$\frac{2x^{-3/6}}{6x^{4/6}} =$

$\frac{x^{-7/6}}{3} =$

$\frac{1}{3x^{7/6}}\left(x^{5/6}\right) = \boxed{\frac{x^{5/6}}{3x^2}}$

26) $3x^4 \cdot 4x^{\frac{3}{2}} \cdot 5x^{-\frac{1}{2}}$

$3x^{8/2} \cdot 4x^{3/2} \cdot 5x^{-1/2} =$

$60x^{10/2} =$

$\boxed{60x^5}$

27) $\left(8p^9\right)^{\frac{2}{3}}$

$8^{2/3}p^{9(2/3)} =$

$\left(\sqrt[3]{8}\right)^2 p^6 =$

$\boxed{4p^6}$

Solve each equation.

28) $\sqrt{5}^{x+1} = 5^{2x+3}$

$5^{1/2(x+1)} = 5^{2x+3}$

$2\left(\frac{1}{2}x + \frac{1}{2}\right) = (2x+3)2$

$x + 1 = 4x + 6$

$-3x = 5$

$\boxed{x = -5/3}$

29) $216^{3x} = 36^{x-2}$

$6^{3(3x)} = 6^{2(x-2)}$

$9x = 2x - 4$

$7x = -4$

$\boxed{x = -\frac{4}{7}}$

30) $\left(\frac{1}{8}\right)^{x} = 16^{2x+3}$

$2^{-3x} = 2^{4(2x+3)}$

$-3x = 8x + 12$

$-11x = 12$

$x = -\frac{12}{11}$

31) $4^{-x+1} \cdot 2^{-3x-4} = 8$

$2^{2(-x+1)} \cdot 2^{-3x-4} = 2^{3}$

$2^{2(-x+1)+(-3x-4)} = 2^{3}$

$-2x + 2 - 3x - 4 = 3$

$-5x - 2 = 3$

$-5x = 5$

$x = -1$

32) $3^{x-2} \cdot \left(\frac{1}{9}\right)^{x+1} = 27^{x}$

$3^{x-2} \cdot 3^{-2(x+1)} = 3^{2x}$

$3^{(x-2)+(-2x-2)} = 3^{2x}$

$x - 2 - 2x - 2 = 2x$

$-x - 4 = 2x$

$-4 = 3x$

$x = -\frac{4}{3}$

33) $7^{2x+3} - 7^{5x} = 0$

$7^{2x+3} = 7^{5x}$

$2x + 3 = 5x$

$3 = 3x$

$x = 1$