

## Average Value (6.2)

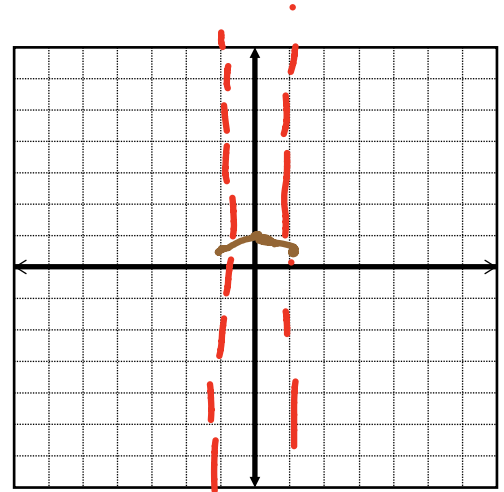
$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx, \quad \text{with the given interval } [a, b]$$

**Average Value:** Calculate the average over the given interval.

1)  $f(x) = \frac{1}{x^2+1}, [-1, 1]$

$$\frac{1}{1+1} \int_{-1}^1 \frac{1}{x^2+1} dx$$

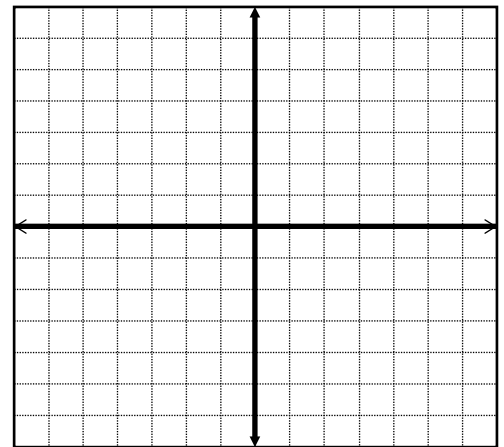
$$\boxed{0.785}$$



2)  $f(x) = \sin(\pi/x), [1, 2]$

$$\frac{1}{2-1} \int_1^2 \sin\left(\frac{\pi}{x}\right) dx$$

$$\boxed{0.374}$$



3) A ball thrown in the air vertically from ground level with initial velocity 18 m/s has height  $h(t) = 18t - 9.8t^2$  at time  $t$  (in seconds). Find the average height and the average speed over the time interval extending from the ball's release to its return to ground level.

$$f(x) = 18x - 9.8x^2 \quad \left[0, \frac{90}{49}\right]$$

$$0 = 18x - 9.8x^2$$

$$0 = x(18 - 9.8x)$$

$$x = \frac{90}{49}$$

$$\frac{49}{90} \int_0^{\frac{90}{49}} (18x - 9.8x^2) dx$$

$$\boxed{5.5 \text{ meters}}$$

## Volumes (6.3)

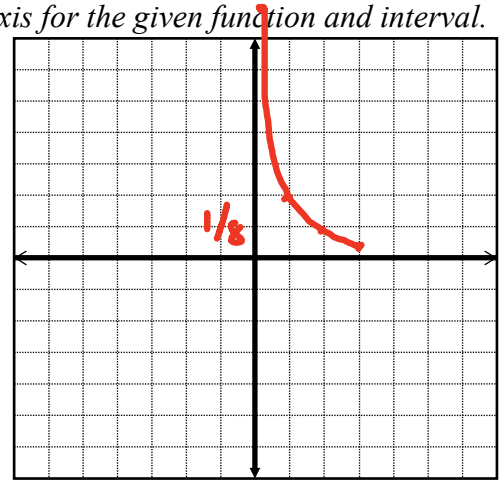
$$V = \pi \int_a^b R^2 dx = \pi \int_a^b f(x)^2 dx$$

**Finding Volumes:** Find the volume of revolution about the x-axis for the given function and interval.

4)  $f(x) = \frac{1}{x^2}, [1, 4]$

$$V = \pi \int_1^4 \left(\frac{1}{x^2}\right)^2 dx$$

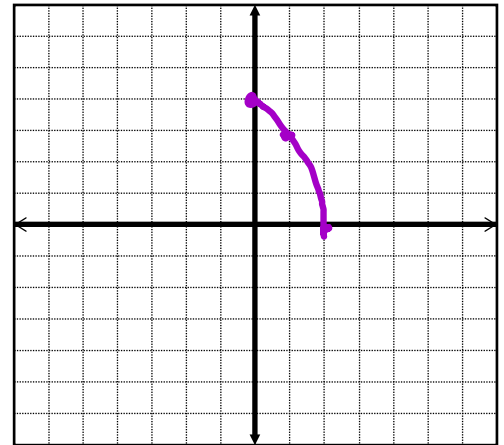
$$\boxed{0.328125\pi}$$



5)  $f(x) = 4 - x^2, [0, 2]$

$$V = \pi \int_0^2 (4 - x^2)^2 dx$$

$$V = \pi \int_0^2 (16 - 8x^2 + x^4) dx$$



6)  $f(x) = \sqrt{x^4 + 1}, [1, 3]$

$$V = \pi \int_1^3 \left[ (x^4 + 1)^{1/2} \right]^2 dx$$

$$V = \pi \int_1^3 (x^4 + 1) dx$$

$$\boxed{50.4\pi}$$

