

Volumes Using Cross Sections (6.2)

$$V = \int_a^b A(x) dx \quad \text{or} \quad V = \int_c^d A(y) dy$$

Finding Volumes Using Cross Sections: Find the volume.

1) Find the volume of the solid whose base is the region bounded by $x = 2 - y^2$ and $x = y^2 - 2$ and whose cross-sections are isosceles triangles with the base perpendicular to the y -axis and the angle between the base and the two sides of equal length is $\pi/4$.

$$A = \frac{1}{2}bh$$

$$b = (2 - y^2) - (y^2 - 2)$$

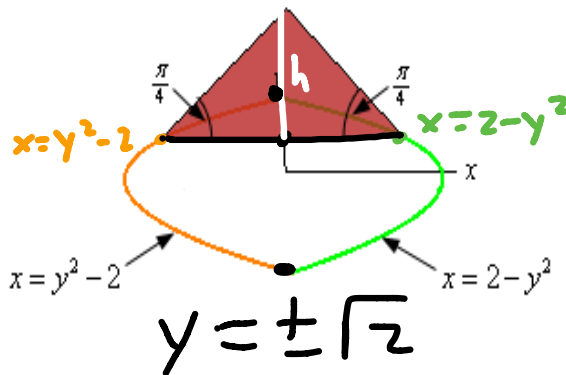
$$b = 2(2 - y^2) = 4 - 2y^2$$

$$\tan \frac{\pi}{4} = h / \frac{b}{2} = \frac{h}{2 - y^2}$$

$$h = 2 - y^2$$

$$V = \int_{-\sqrt{2}}^{\sqrt{2}} \frac{1}{2} (4 - 2y^2) (2 - y^2) dy$$

$$V = \int_{-\sqrt{2}}^{\sqrt{2}} (2 - y^2)^2 dy \approx \boxed{6.034}$$



2) Find the volume between $f(x) = e^x$ and $g(x) = \ln|x+2| + 2$, with square cross-sectional shapes that are perpendicular to the x -axis.

Using a calculator the intersections for $x = -1.841, 1.146$.

$$A = x^2$$

$$x = \ln|x+2| + 2 - e^x$$

$$V = \int_{-1.841}^{1.146} (\ln|x+2| + 2 - e^x)^2 dx$$

$$\boxed{V \approx 5.82}$$