

## AP Calculus FINAL REVIEW

Evaluate the following limits:

1.  $\lim_{x \rightarrow 6} \frac{x+6}{x^2-6}$

2.  $\lim_{x \rightarrow 3} \frac{x^2-2x}{x+1}$

3.  $\lim_{x \rightarrow \infty} \frac{5x^2+7}{3x^2-x}$

4.  $\lim_{x \rightarrow \infty} \frac{x-2}{\sqrt{x^2+2x+1}}$

5.  $\lim_{x \rightarrow 3} f(x)$  when  $f(x) = \begin{cases} x-1, & x \leq 3 \\ 3x-7, & x > 3 \end{cases}$

6.  $\lim_{x \rightarrow 2} \frac{x^4-16}{x+2}$

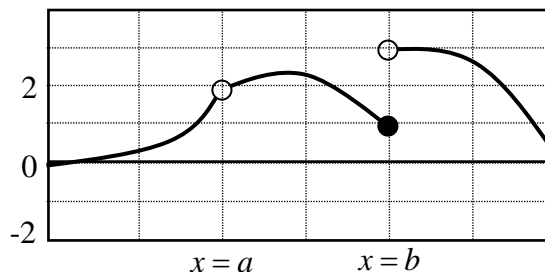
7.  $\lim_{x \rightarrow 16} \frac{\sqrt{x}-4}{16-x}$

8. Find  $\lim_{t \rightarrow \infty} \frac{3t^2+2}{2+t^2-3t^3}$

9. The graph of  $f(x)$  is shown

a. What is the  $\lim_{x \rightarrow a} f(x)$ ?

b. What is the  $\lim_{x \rightarrow b} f(x)$ ?



10. For what value of the constant  $c$  is the function  $f$  continuous on  $(-\infty, \infty)$ ?

$$f(x) = \begin{cases} cx+8, & x \leq 4 \\ cx^2-4, & x > 4 \end{cases}$$

11.  $f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x+k, & x > 2 \end{cases}$  Find the value of  $k$  that will make  $f(x)$  continuous.

12. Which of the following are not continuous everywhere?

a)  $y = \frac{1}{x^2}$     b)  $y = |5-x^2|$     c)  $y = \frac{x^2-9}{x^2-5x+6}$     d)  $y = \sqrt{x^2-16}$

**Find the derivatives of the following:**

**13.**  $f(x) = 4x^3 - 3x^2 + 8$

**14.**  $f(x) = \sqrt{x} + \frac{1}{x}$

**15.**  $f(x) = \frac{4x+1}{x^2-5}$

**16.**  $f(x) = (x^3 + 2x)^{27}$

**17.**  $f(x) = 2x\sqrt{3x-1}$

**18.**  $f(x) = x^3 \sin^2(5x)$

**19.**  $f(x) = \sqrt{\cos(5x)}$

**20.**  $f(y) = e^{\cos^4(3y)}$

**21.**  $\frac{d}{dx}(\sin^3(x^2)) =$

**22.** Find  $\frac{d^2y}{dx^2}$  if  $y = \frac{x+1}{x}$

**23.** Find  $\frac{d^2y}{dx^2}$  if  $y = \sin(3x^2)$

**24.** Find  $\frac{dy}{dx}$  for  $7y^4 + x^3y + x = 4$

**25.** Find  $\frac{dy}{dx}$  for  $y^2 - 3xy + 2x^2 = 4$  at  $(3, 2)$ .

**26.** If  $y^3 - 3xy = 12$  then  $\frac{dy}{dx} = ?$

**27.** If  $\sin(x + y) = x$ , then  $\frac{dy}{dx} = ?$

28. If  $xy^3 + 3xy^2 = 7$ , then at the point  $(2,1)$ ,  $y' = ?$

29. If  $y^3 - 5x = 4$  then  $\frac{d^2y}{dx^2} = ?$

30. If  $3x^2 - 4y^2 = 8$  what is the value of  $\frac{d^2y}{dx^2}$  at the point  $(2,1)$ ?

31. What is the slope of the curve  $y^2 - xy^3 = 12$  at the point  $y = 2$ ?

32. If  $f(x) = -x^2 + 3x - \frac{1}{x}$ , then  $f'(-1) = ?$

33. What point(s) on the graph of  $y = \frac{3}{2}x^2$  is/are parallel to the line  $3x - 2y = 12$ ?

34. If  $f(x) = x^3 - 3x^2 - 5x - 1$ , when is the slope of the tangent line of  $f(x)$  parallel to the line  $2x - 4y = 5$ ?

35. Find the equation of the tangent line of  $y = \sin(3x)$  at  $x = \frac{\pi}{3}$ .

36. Find an equation of the line that is tangent to  $y = \frac{1-x}{1+x}$  at the point  $x = 2$ .

37. Find an equation of the line that is tangent to  $y = x \cos(3x)$  at the point  $x = \pi$ .

38. At which point(s) does the graph of  $y = \frac{x}{x^2 + 9}$  have a horizontal tangent line?

39. At which point(s) does the graph of  $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x$  have a horizontal tangent line?

40. Let  $f(x) = \sqrt{x}$ . If the rate of change of  $f$  at  $x = c$  is twice the rate of change at  $x = 4$ , then what does  $c$  equal?

41. What does  $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$  represent?

42. Find  $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$

Use the following table for 43 - 45.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 0   | 2      | 0       | 1      | 1       |
| 1   | 3      | 2       | 3      | 2       |
| 2   | 4      | 1       | -5     | 1       |

43. If  $h(x) = f(x) \cdot g(x)$ , find  $h'(1) =$

44. If  $h(x) = f \circ g(x)$ , find  $h'(0) =$

45. If  $h(x) = \frac{f(x)}{g(x)}$ , find  $h'(2) =$

For 46-47 use  $f(x) = -x^3 + 12x + 5$ .

46. Find the intervals on which  $f(x)$  is increasing or decreasing.

47. Find the relative max and the relative min of  $f(x)$ .

For 48-49 use  $f(x) = x^3 - 3x^2 + 1$ .

48. Find the intervals on which  $f(x)$  is concave up or concave down?

49. Find the inflection points of  $f(x)$ .

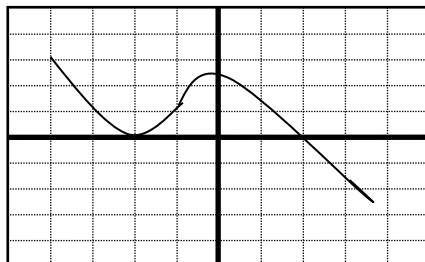
50. The graph of  $y = x^4 - 4x^3 + 6x^2 + 5$  is concave down over what interval(s)?

51. Where ( $x = ?$ ) does the graph of the function  $y = x^3 - 3x^2 + 2x - 3\sin(x)$  change concavity?

52. The graph of function  $f$  is shown at the right. If the function  $G$  is defined by

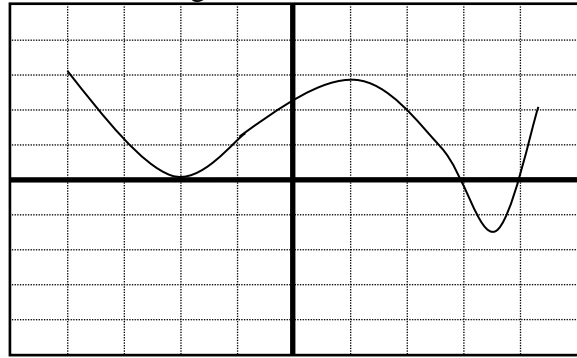
$$G(x) = \int_{-3}^x f(t) dt \text{ for } -3 \leq x \leq 3, \text{ which of the following is true?}$$

- a.  $G$  is increasing on  $(0, 2)$
- b.  $G$  is decreasing on  $(-3, -2)$
- c.  $G(0) < 0$

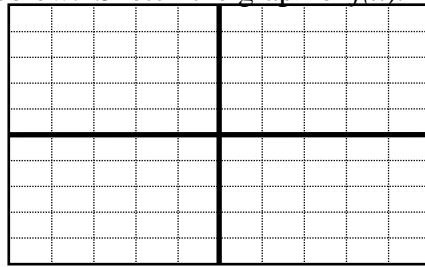
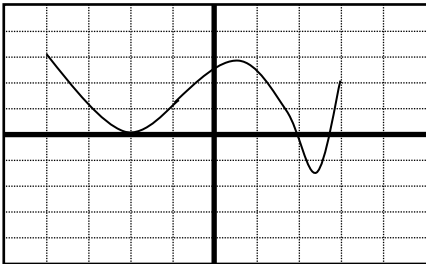


53. The graph of  $f'$  (the derivative of a function,  $f$ ) is shown at the right

- $f$  is increasing on what interval(s)?
- Where are the points of inflection?
- When is  $f$  concave downward?



54. The graph of the derivative of  $f(x)$  is shown below. Sketch the graph of  $f(x)$ .



55. Find the absolute max and min values of  $f(x) = 2x^3 - 3x^2 - 12x$  on  $[-2, 3]$ .

56. Find the value of  $c$  which satisfies the Mean Value Theorem for  $f(x) = x^2 + x$  on  $[-4, 6]$ .

57. A man is standing fifty feet from an intersection. A car is going down the road that is a right angle to the intersection at a rate of 40 feet per second. How fast is the car moving away from the man 3 seconds after it passes through the intersection?

58. If  $y = 3x - 5$  what is the minimum product of  $xy$ ?

Find the integrals of the following:

59.  $\int (4x^2 - 8x + 3)dx$

60.  $\int \frac{9}{x^5} dx$

61.  $\int x^{-3/5} dx$

62.  $\int \sqrt[5]{x^3} dx$

63.  $\int 5x \cos(x^2) dx$

64.  $\int \frac{x^2 dx}{\sqrt{x^3 + 9}}$

65.  $\int \frac{2x^4 - x + 2}{x^3} dx$

66.  $\int \frac{x^2}{x^3 - 4} dx$

67.  $\int x^2 e^{-2x^3} dx$

68.  $\int_1^2 (4x - 2)^3 dx$

69.  $\int_{-1}^4 |x - 1| dx$

70.  $\int_{-\pi/2}^{\pi/2} (1 + \cos x) dx$

71.  $\int_{-1}^0 \frac{x}{x^2 + 5} dx$

72.  $\frac{d^2 y}{dx^2} = 0; \quad y'(0) = 2$   
 $y(0) = 0$

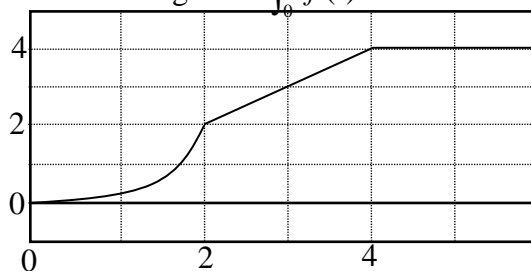
73. Solve the initial value problem:  $\frac{dy}{dx} = 3x^2 - 2x + 5$ ,  $y(1) = -2$ .

74. Evaluate  $\int_{-2}^3 f(x)dx$ , where  $f(x) = \begin{cases} -x, & x \geq 0 \\ x^2, & x < 0 \end{cases}$ .

75. If  $\int_2^9 f(x)dx = 12$  and  $\int_4^2 f(x)dx = 2$ , find  $\int_4^9 f(x)dx$ .

76. If  $\int_a^b f(x)dx = 3a + 2b$  then  $\int_a^b [f(x) + 2]dx = ?$

77. The graph of  $f$  is shown at the right. If  $\int_0^2 f(t)dt = 1.2$  and  $F'(x) = f(x)$ , find  $F(6) - F(0)$ .



78. If  $F(x) = \int_3^x \sin^3(3t)dt$  then  $F'\left(\frac{\pi}{2}\right) = ?$



79. If  $F(x) = \int_0^x \sqrt{\sin t} dt$ , then  $F'(0.5) = ?$

80. Consider the function  $F$  defined so that  $F(x) + 6 = \int_3^x \tan\left(\frac{\pi t}{4}\right) dt$ . Find  $F(3) + F'(3)$ .

**Be sure to go through all of the test reviews as well – practice, practice, practice!**

**Final Review Solutions**

1.  $\frac{2}{5}$

2.  $\frac{3}{4}$

3.  $\frac{5}{3}$

4. 1

5. 2

6. 0

7.  $-\frac{1}{8}$

8. 0

9a) 2 b) DNE

10.  $c = 1$

11.  $\frac{4}{3}$

12. a, c and d

13.  $12x^2 - 6x$

14.  $\frac{1}{2\sqrt{x}} - \frac{1}{x^2}$

15.  $\frac{-4x^2 - 2x - 20}{(x^2 - 5)^2}$

16.  $27(x^3 + 2x)^{26} (3x^2 + 2)$

17.  $\frac{6x}{2\sqrt{3x-1}} + 2\sqrt{3x-1} = \frac{9x-2}{\sqrt{3x-1}}$

18.  $3x^2 \sin^2 5x + 10x^3 \sin 5x \cos 5x$

19.  $\frac{-5 \sin 5x}{2\sqrt{\cos 5x}}$

20.  $-12 \sin(3y) \cos^3(3y) e^{\cos^4(3y)}$

21.  $6x \sin^2(x^2) \cos(x^2)$

22.  $\frac{2}{x^3}$

23.  $6 \cos(3x^2) - 36x^2 \sin(3x^2)$

24.  $\frac{-1 - 3x^2 y}{28y^3 + x^3}$

25.  $\frac{6}{5}$

26.  $\frac{3y}{3y^2 - 3x}$

27.  $\sec(x + y) - 1$

28.  $-\frac{2}{9}$

29.  $\frac{-50}{9y^5}$

30.  $-\frac{3}{2}$

31.  $\frac{1}{2}$

32. 6

33.  $\left(\frac{1}{2}, \frac{3}{8}\right)$

34.  $x = 2.683$  and  $-0.683$

35.  $y = -3x + \pi$

36.  $y + \frac{1}{3} = -\frac{2}{9}(x-2)$

37.  $y = -x$

38.  $\left(3, \frac{1}{6}\right)$  and  $\left(-3, -\frac{1}{6}\right)$

39.  $\left(2, \frac{2}{3}\right)$  and  $\left(1, \frac{5}{6}\right)$

40.  $c = 1$

41.  $f'(3)$

42.  $\cos x$

43. 12

44. 2

45.  $-\frac{9}{25}$

46. increasing:  $(-2, 2)$   
decreasing:  
 $(-\infty, -2), (2, \infty)$

47. max:  $(2, 21)$   
min:  $(-2, -11)$

48. up:  $(1, \infty)$   
down:  $(-\infty, 1)$

49.  $(1, -1)$

50. never CD

51.  $(1, -4)$

52. a

53. a)  $(-4, -2) \cup (-2, 3) \cup (4, 4.5)$

b)  $x = -2, 1, 3.5$

c)  $(-4, -2) \cup (1, 3.5)$

54. Graph

55. max value = 7  
min value = -20

56.  $c = 1$

57. 36.923 ft/sec

58.  $-25/12$

59.  $\frac{4x^3}{3} - 4x^2 + 3x + C$

60.  $-\frac{9}{4x^4} + C$

61.  $\frac{5x^{2/5}}{2} + C$

62.  $\frac{5x^{8/5}}{8} + C$

63.  $\frac{5}{2}\sin(x^2) + C$

64.  $\frac{2}{3}\sqrt{x^3+9} + C$

65.  $x^2 + \frac{1}{x} - \frac{1}{x^2} + C$

66.  $\frac{1}{3}\ln|x^3-4| + C$

67.  $-\frac{1}{6}e^{-2x^3} + C$

68. 80

69.  $13/2$

70.  $2 + \pi$

71.  $\frac{1}{2}\ln\frac{5}{6}$

72.  $y = 2x$

73.  $y = x^3 - x^2 + 5x - 7$

74.  $-\frac{11}{6}$

75. 14

76.  $a + 4b$

77. 15.2

78. -1

79. 0.692

80. -7