AP Calculus
Solids with known cross-sections

1.

Find the volume of the solid whose base is the region bounded between the curves $y = x$ and $y = x^2$, and whose cross sections perpendicular to the *x*-axis are squares.

2.

The base of a certain solid is the region enclosed by $y = \sqrt{x}, y = 0,$ and $x = 4$. Every cross section perpendicular to the *x*-axis is a semicircle with is diameter across the base. Find the volume of the solid.

3.

Consider the region enclosed between $y = \sqrt{x}, x = 1, x = 4,$ and the *x*-axis Find the volume of the solid that is formed when the enclosed region is revolved about the *x*-axis.

4.
The base of a solid in the *xy*-plane is a right triangle bounded by the axes and $x + y = 2$. Cross sections of the solid perpendicular to the *x*-axis are squares. Find the volume.

5.

The base of a solid is the circle $x^2 + y^2 = 9$. Cross sections of the solid perpendicular to the *x*-axis are semicircles. Find the volume of the solid.

6.

The base of a solid is the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$, $y = e^{-3x}$ and the vertical line $x = 1$. For this solid each cross section perpendicular to the *x*-axis is a rectangle whose height is 5 times its length of its base. Find the volume of the solid. **CALCULATOR!!**

7.

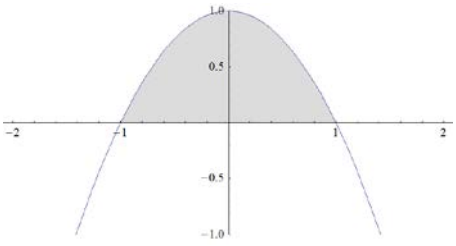
3. The base of a solid is bound by $y = x^3, y = 0,$ and $x = 1$, Find the volume of the solid that has cross sections that are equilateral triangles taken perpendicular to the y-axis.

8.

Find the volume of the solid with circular base of diameter 10 cm and whose cross-sections perpendicular to a given diameter are equilateral triangles.

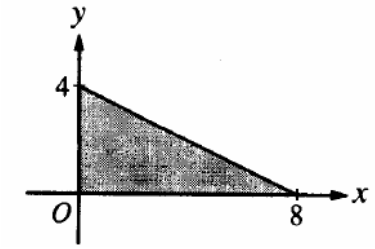
9.

The base of a solid is the region bounded by the graph of $y = 1 - x^2$ and the *x*-axis. For this solid, each cross section perpendicular to the *x*-axis is a rectangle with height three times the base. What is the volume of this solid?



10.

The base of a solid is the region in the first quadrant bounded by the *x*-axis, the *y*-axis, and the line $x + 2y = 8$, as shown in the figure. If cross sections of the solid perpendicular to the *x*-axis are semicircles, what is the volume of the solid?

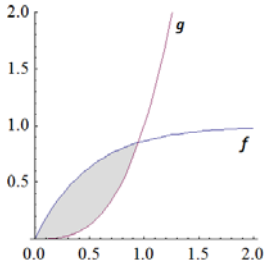


11.

The region bounded by the graph of $y = 2x - x^2$ and the *x*-axis is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is an equilateral triangle. What is the volume of the solid?

12.

The region in Quadrant I bounded by the graph of $f(x) = 1 - e^{-x}$ and $g(x) = x^3$ is the base of a solid. Find the volume of this solid, if



(a) For this solid, each cross section perpendicular to the *x*-axis is an isosceles right triangle with one leg across the base of the solid.
(b) For this solid, each cross section perpendicular to the *x*-axis is an isosceles right triangle with the hypotenuse across the base of the solid.

13.

Let *R* be the region in Quadrant I bounded by the graph of $y = e^x$, the *y*-axis, and the horizontal line $y = 4$.

(a) Find the area of *R*.
(b) The region *R* is the base of a solid. For this solid, each cross section **perpendicular to the *y*-axis** is a square. Find the volume of this solid.

14.

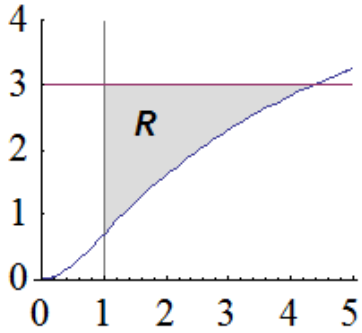
The base of a solid is the region in the first quadrant bounded by the *y*-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the *x*-axis is a square. What is the volume of the solid?

15.

Find the volume of the solid with base given by the ellipse $9x^2 + 4y^2 = 36$ and whose cross sections perpendicular to the major axis are squares.

16.

Let *R* be the region bounded by the graph of $y = \ln\left(x^2 + 1\right)$, the horizontal line $y = 3$, and the vertical line $x = 1$, as shown in the figure.



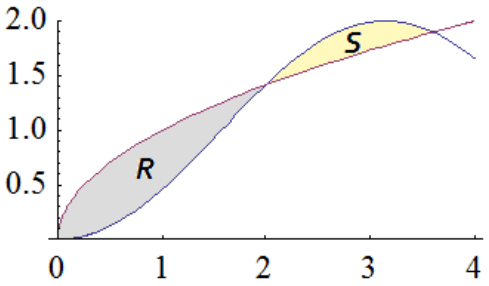
(a) Find the area of *R*.

(b) The region *R* is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is a triangle with height equal to twice the length of the base. Find the volume of this solid.

(c) Another solid whose base is also the region *R*. For this solid, each cross section perpendicular to the *x*-axis is a semicircle with diameter across the base. Find the volume of this solid.

17.

Let *R* and *S* be the regions bounded by the graphs of $f(x) = 1 - \cos x$ and $g(x) = \sqrt{x}$ in Quadrant I.



(c) Find the total area of the regions bounded by *f* and *g* in Quadrant I, that is, $R + S$.

(d) Region *R* is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is an equilateral triangle. Find the volume of this solid.

(e) Region *S* is the base of another solid. For this solid, each cross section perpendicular to the *x*-axis is a semicircle. Find the volume of this solid.

18.

The region in Quadrant I bounded by the graphs of $y = \tan^{-1} x$ and $y = \frac{1}{2}x$ is the base of a solid. For this solid, each cross section **perpendicular to the *y*-axis** is a rectangle with height four times the length of the width. Find the volume of this solid.

Answers

1. 1/30	2. π	3. 15π/2	4. 8/3	5. 18π	6. 1.554
7. 0.043	8. 288.675	9. 3.2	10. 16.755	11. 0.462	12. (a) 0.016 (b) 0.008
13. (a) 2.545 (b) 2.597		14. 6.612	15. 64	16. (a) 3.310 (b) 4.722 (c) 1.854	
17. (a) 1.077 (b) 0.165 (c) 0.024		18. 0.770			