

## 2.2 Limits: A Numerical and Graphical Approach

### Limits

Suppose that  $f$  is a function defined on some open interval containing the number  $a$ . The function  $f$  may or may not be defined at  $a$ .

Limit notation  $\lim_{x \rightarrow a} f(x) = L$  is read “the limit of  $f(x)$  as  $x$  approaches  $a$  equals the number  $L$ .” This means as  $x$  gets closer to  $a$ , but remains unequal to  $a$ , the corresponding values of  $f(x)$  get closer to  $L$ .

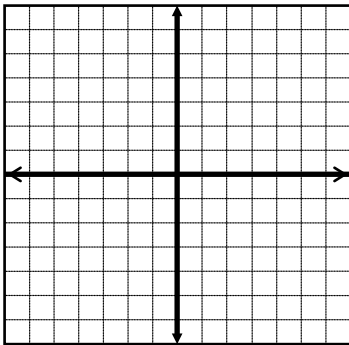
**Finding a Limit Using a Table:** Construct a table to find the indicated limit.

1)  $\lim_{x \rightarrow 0} \frac{x+1}{x^2+1}$

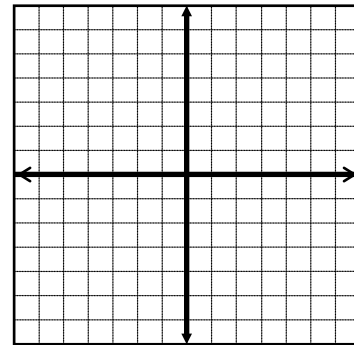

2)  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$


**Graphing a Function to Find the Limit:** Graph the function. Then use your graph to find the limit.

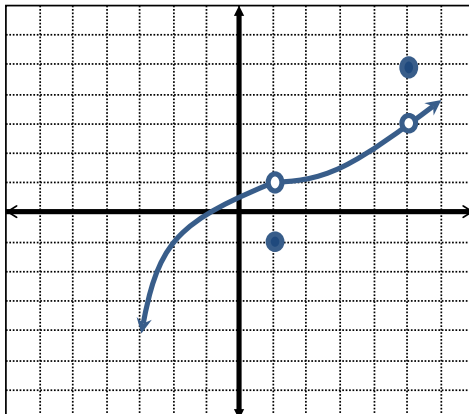
3)  $f(x) = \frac{x-2}{x^2-4}$ ,  $\lim_{x \rightarrow 2} f(x)$



4)  $f(x) = \begin{cases} 3x-1 & \text{if } x < 1 \\ -x+1 & \text{if } x \geq 1 \end{cases}$ ,  $\lim_{x \rightarrow 1} f(x)$



**Finding a Limit Using a graph:** Use the graph of  $f$  to find the indicated limit and function value.



5)  $\lim_{x \rightarrow 1} f(x)$

6)  $f(1)$

7)  $\lim_{x \rightarrow -2} f(x)$

8)  $f(-2)$

9)  $\lim_{x \rightarrow 5} f(x)$

10)  $f(5)$

## Equal and Unequal One-Sided Limits

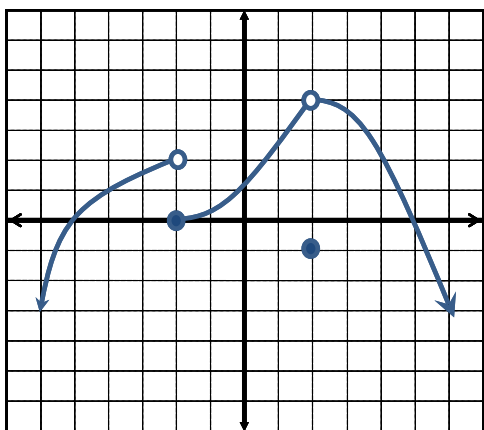
### Equal One-Sided Limits:

$\lim_{x \rightarrow a} f(x) = L$  if and only if both  $\lim_{x \rightarrow a^-} f(x) = L$  and  $\lim_{x \rightarrow a^+} f(x) = L$ .

### Unequal One-Sided Limits:

If  $\lim_{x \rightarrow a^-} f(x) = L$  and  $\lim_{x \rightarrow a^+} f(x) = M$ , where  $L \neq M$ , then  $\lim_{x \rightarrow a} f(x)$  does not exist.

**One-Side Limits:** The graph of a function  $f$  is given. Use the graph to find the indicated limits and function values, or state that a limit or function value does not exist.



11)  $\lim_{x \rightarrow -2^-} f(x)$

12)  $\lim_{x \rightarrow -2^+} f(x)$

13)  $\lim_{x \rightarrow -2} f(x)$

14)  $f(-2)$

15)  $\lim_{x \rightarrow 2^-} f(x)$

16)  $\lim_{x \rightarrow 2^+} f(x)$

17)  $\lim_{x \rightarrow 2} f(x)$

18)  $f(2)$

19)  $\lim_{x \rightarrow 5^-} f(x)$

20)  $\lim_{x \rightarrow 5^+} f(x)$

21)  $\lim_{x \rightarrow 5} f(x)$

22)  $f(5)$