

Rolle's Theorem and The Mean Value Theorem (4.3)

Rolle's Theorem

Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval $[a, b]$
2. f is differentiable on the open interval (a, b)
3. $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$

Verify that the function satisfies the three hypotheses of **Rolle's Theorem** on the given interval. Then find all numbers c that satisfy the conclusion of **Rolle's Theorem**.

1. $f(x) = 2x^3 - 2x + 1$ $[-1, 1]$

2. $f(x) = 2x\sqrt{x+4}$ $[-8, -4]$

$f(x) = 2x\sqrt{x+4}$ $[-4, 0]$

Mean Value Theorem

Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a, b]$
2. f is differentiable on the open interval (a, b)

Then there is a number c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

Verify that the function satisfies the hypotheses of the **Mean Value Theorem** on the given interval. Then, find all numbers c that satisfy the conclusion of the **Mean Value Theorem**.

3. $f(x) = 2x^3 - 5x^2 + 1$ $[0, 2]$

4. $f(x) = \frac{2x}{x-2}$ $[0, 4]$

$f(x) = \frac{2x}{x-2}$ $[3, 6]$