Name ____ Per. _____ Date_________

5.1,5.2 (Riemann Sums/The Definite Integral) Notes 23

Riemann Sum	$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x dx$ $\Delta x = \frac{b-a}{n}$; $n =$ the number of subintervals
Fundamental Theorem of Calculus (Part 1)	$F(x) = \int_a^b f(x)dx = F(b) - F(a)$

Divide the interval [a,b] into n = 3 subintervals of equal length and compute the sum of the areas (a) using the right hand sum, (b) the left hand sum, c) the midpoint rule, and d) the integral definition.

1) $\int_0^6 (x^2 - 4)\,dx$

RHS: $\frac{6-0}{3}\left(f(2)+f(4)+f(6)\right)$

$2(0+12+32) = \boxed{88}$

LHS: $\frac{6-0}{3}\left(f(0)+f(2)+f(4)\right)$

$2(-4+0+12) = \boxed{16}$

MP: $\frac{6-0}{3}\left(f(1)+f(3)+f(5)\right)$

$2(-3+5+21) = \boxed{46}$

FTC: $\int_0^6 (x^2-4)\,dx$

$\frac{x^3}{3} - 4x \Big|_0^6$

$\left(\frac{6^3}{3} - 4(6)\right) - \left(\frac{0^3}{3} - 4(0)\right) = \boxed{48}$

Divide the interval [a,b] into n = 4 subintervals of equal length and compute the sum of the areas (a) using the right hand sum, (b) the left hand sum, c) the midpoint rule, and d) the integral definition.

2) $\int_1^3 \left(-x^3+4x-2\right)dx$

RHS: $\frac{3-1}{4}\left(f(1.5)+f(2)+f(2.5)+f(3)\right)$

$\frac{1}{2}\left(0.625+-2+-7.625+-17\right)=\boxed{-13}$

LHS: $\frac{3-1}{4}\left(f(1)+f(1.5)+f(2)+f(2.5)\right)$

$\frac{1}{2}\left(1+0.625-2-7.625\right)=\boxed{-4}$

MP: $\frac{3-1}{4}\left(f(1.25)+f(1.75)+f(2.25)+f(2.75)\right)$

$\boxed{-7.75}$

FTC: $\int_1^3\left(-x^3+4x-2\right)dx$

$-\frac{x^4}{4}+2x^2-2x\Big|_1^3$

$\left(-\frac{81}{4}+18-6\right)-\left(-\frac{1}{4}+\cancel{2}-\cancel{2}\right)$

$-20+12=\boxed{-8}$