# 5.6 Integration by Substitution

# Integration by Substitution

1. $$\int \frac{\left(1+\sqrt{x}\right)^3}{\sqrt{x}}\,dx$$

$$\int \frac{u^3}{\sqrt{x}}dx \qquad u = 1+\sqrt{x} \qquad du = \frac{1}{2\sqrt{x}}dx$$

$$\int \frac{u^3}{\sqrt{x}} \bullet 2\sqrt{x}du \qquad 2\sqrt{x}du = dx$$

$$\int 2u^3 du$$

$$\frac{1}{2}u^4 + C$$

$$\boxed{\frac{1}{2}\left(1+\sqrt{x}\right)^4 + C}$$

# Integration by Substitution

2. $\int (x^4 - 2x^2 + 8x - 2)(x^3 - x + 2)\, dx$

$$\int u (x^3 - x + 2)\, dx \qquad u = x^4 - 2x^2 + 8x - 2$$

$$\int u (x^3 - x + 2) \bullet \frac{du}{4(x^3 - x + 2)} \qquad du = (4x^3 - 4x + 8)\, dx$$

$$\int \frac{u}{4}\, du \qquad \frac{du}{4(x^3 - x + 2)} = dx$$

$$\frac{u^2}{8} + C$$

$$\boxed{\frac{(x^4 - 2x^2 + 8x - 2)^2}{8} + C}$$

# Substitution in Definite Integrals

3. $\int_{-1}^{1} t^3 \left(1+t^4\right)^3 dt$

$u = 1 + t^4$

$\int_{-1}^{1} t^3 u^3 dt$

$du = 4t^3 dt$

$\int_{-1}^{1} t^3 u^3 \bullet \frac{du}{4t^3}$

$\frac{du}{4t^3} = dt$

$\int_{-1}^{1} \frac{u^3}{4} du$

$\text{UL}=1+1^4 = 2$

$\text{LL}=1+\left(-1\right)^4 = 2$

$\int_{2}^{2} \frac{u^3}{4} du$

$\left.\frac{u^4}{16}\right|_{2}^{2}$

$F\left(2\right) - F\left(2\right) = \frac{16}{16} - \frac{16}{16} = \boxed{0}$

# Substitution in Definite Integrals

4. $$\int_1^4 \frac{10\sqrt{v}}{\left(1+v^{3/2}\right)^2}dv$$

$$u = 1 + v^{3/2}$$

$$du = \frac{3}{2}\sqrt{v}\,dv$$

$$\frac{2du}{3\sqrt{v}} = dv$$

$$\int_1^4 \frac{10\sqrt{v}}{u^2}dv$$

$$\int_1^4 \frac{10\sqrt{v}}{u^2} \bullet \frac{2du}{3\sqrt{v}}$$

$$\int_1^4 \frac{20}{3u^2}du$$

$$\text{UL}=1+4^{3/2}=9$$

$$\text{LL}=1+1^{3/2}=2$$

$$\int_2^9 \frac{20}{3}u^{-2}du$$

$$-\frac{20}{3u}\Bigg|_2^9 \qquad F(9)-F(2) = -\frac{20}{27}-\left(-\frac{10}{3}\right) = \boxed{\frac{70}{27}}$$

# Substitution in Definite Integrals

5. $\int \cos(3z+4)\,dz$

$u = 3z+4$

$\int \cos(u)\,dz$

$du = 3dz$

$\int \cos u \frac{du}{3}$

$\frac{du}{3} = dz$

$\frac{1}{3}\int \cos u\,du$

$\frac{\sin u}{3} + C$

$$\frac{\sin(3z+4)}{3} + C$$

# Substitution in Definite Integrals

6. $\int \frac{dx}{\sqrt{5x+8}}$

$u = 5x + 8$

$\int (5x+8)^{-1/2} dx$

$du = 5dx$

$\int u^{-1/2} dx$

$\frac{du}{5} = dx$

$\int u^{-1/2} \left( \frac{du}{5} \right)$

$\frac{2u^{1/2}}{5} + C$

$\boxed{\frac{2(5x+8)^{1/2}}{5} + C}$

# Substitution in Definite Integrals

7. $$\int_{-2}^{0} \frac{9r^2\,dr}{\sqrt{1-r^3}}$$

$$u = 1 - r^3$$

$$\int_{-2}^{0} \frac{9r^2\,dr}{\sqrt{u}}$$

$$du = -3r^2\,dr$$

$$-\frac{du}{3r^2} = dr$$

$$\int_{-2}^{0} \frac{9r^2}{\sqrt{u}}\left(-\frac{du}{3r^2}\right)$$

$$\int_{9}^{1} -3u^{-1/2}\,du$$

$$-6\sqrt{u}\ \Big|_{9}^{1} \qquad F(1) - F(9) = \quad -6 + 18 = \boxed{12}$$

# Substitution in Definite Integrals

8. $\int_0^{\pi/2} \sin^5(3x)\cos(3x)\,dx$

$u = \sin(3x)$

$du = 3\cos(3x)\,dx$

$\frac{du}{3\cos(3x)} = dx$

$\int_0^{\pi/2} u^5 \cos(3x)\,dx$

$\int_0^{\pi/2} u^5 \cos(3x)\left(\frac{du}{3\cos(3x)}\right)$

$\int_0^{-1} \frac{u^5}{3}\,du$

$\frac{u^6}{18}\Big|_0^{-1}$

$F(-1) - F(0) = \frac{1}{18} - 0 = \boxed{\frac{1}{18}}$