

5.6 Integration by Substitution

Integration by Substitution

$$1. \int \frac{(1 + \sqrt{x})^3}{\sqrt{x}} dx$$

$$\int \frac{u^3}{\sqrt{x}} dx$$

$$u = 1 + \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$\int \frac{u^3}{\sqrt{x}} \cdot 2\sqrt{x} du$$

$$2\sqrt{x} du = dx$$

$$\int 2u^3 du$$

$$\frac{1}{2} u^4 + C$$

$$\frac{1}{2} (1 + \sqrt{x})^4 + C$$

Integration by Substitution

$$2. \int (x^4 - 2x^2 + 8x - 2)(x^3 - x + 2) dx$$

$$\int u(x^3 - x + 2) dx$$

$$u = x^4 - 2x^2 + 8x - 2$$

$$\int u(x^3 - x + 2) \cdot \frac{du}{4(x^3 - x + 2)}$$

$$du = (4x^3 - 4x + 8) dx$$

$$\int \frac{u}{4} du$$

$$\frac{du}{4(x^3 - x + 2)} = dx$$

$$\frac{u^2}{8} + C$$

$$\frac{(x^4 - 2x^2 + 8x - 2)^2}{8} + C$$

Substitution in Definite Integrals

$$3. \int_{-1}^1 t^3 (1+t^4)^3 dt$$

$$u = 1 + t^4$$

$$\int_{-1}^1 t^3 u^3 dt$$

$$du = 4t^3 dt$$

$$\int_{-1}^1 t^3 u^3 \cdot \frac{du}{4t^3}$$

$$\frac{du}{4t^3} = dt$$

$$\int_{-1}^1 \frac{u^3}{4} du$$

$$UL = 1 + 1^4 = 2$$

$$\int_2^2 \frac{u^3}{4} du$$

$$LL = 1 + (-1)^4 = 2$$

$$\frac{u^4}{16} \Big|_2^2$$

$$F(2) - F(2) = \frac{16}{16} - \frac{16}{16} = 0$$

Substitution in Definite Integrals

$$4. \int_1^4 \frac{10\sqrt{v}}{\left(1+v^{3/2}\right)^2} dv$$

$$\int_1^4 \frac{10\sqrt{v}}{u^2} dv$$
$$\int_1^4 \frac{10\sqrt{v}}{u^2} \cdot \frac{2du}{3\sqrt{v}}$$

$$\int_1^4 \frac{20}{3u^2} du$$

$$\int_2^9 \frac{20}{3} u^{-2} du$$

$$-\frac{20}{3u} \Big|_2^9$$

$$u = 1 + v^{3/2}$$

$$du = \frac{3}{2} \sqrt{v} dv$$

$$\frac{2du}{3\sqrt{v}} = dv$$

$$UL = 1 + 4^{3/2} = 9$$

$$LL = 1 + 1^{3/2} = 2$$

$$F(9) - F(2) = -\frac{20}{27} - \left(-\frac{10}{3}\right) = \frac{70}{27}$$

Substitution in Definite Integrals

$$5. \int \cos(3z + 4) dz$$

$$u = 3z + 4$$

$$\int \cos(u) dz$$

$$du = 3dz$$

$$\int \cos u \frac{du}{3}$$

$$\frac{du}{3} = dz$$

$$\frac{1}{3} \int \cos u du$$

$$\frac{\sin u}{3} + C$$

$$\frac{\sin(3z + 4)}{3} + C$$

Substitution in Definite Integrals

$$6. \int \frac{dx}{\sqrt{5x+8}}$$

$$u = 5x + 8$$

$$\int (5x+8)^{-1/2} dx$$

$$du = 5dx$$

$$\int u^{-1/2} dx$$

$$\frac{du}{5} = dx$$

$$\int u^{-1/2} \left(\frac{du}{5} \right)$$

$$\frac{2u^{1/2}}{5} + C$$

$$\boxed{\frac{2(5x+8)^{1/2}}{5} + C}$$

Substitution in Definite Integrals

$$7. \int_{-2}^0 \frac{9r^2 dr}{\sqrt{1-r^3}}$$

$$u = 1 - r^3$$

$$\int_{-2}^0 \frac{9r^2 dr}{\sqrt{u}}$$

$$du = -3r^2 dr$$

$$-\frac{du}{3r^2} = dr$$

$$\int_{-2}^0 \frac{9r^2}{\sqrt{u}} \left(-\frac{du}{3r^2} \right)$$

$$\int_9^1 -3u^{-1/2} du$$

$$-6\sqrt{u} \Big|_9^1$$

$$F(1) - F(9) = -6 + 18 = \boxed{12}$$

Substitution in Definite Integrals

$$8. \int_0^{\pi/2} \sin^5(3x) \cos(3x) dx \quad u = \sin(3x)$$

$$\int_0^{\pi/2} u^5 \cos(3x) dx \quad du = 3 \cos(3x) dx$$

$$\frac{du}{3 \cos(3x)} = dx$$

$$\int_0^{\pi/2} u^5 \cos(3x) \left(\frac{du}{3 \cos(3x)} \right)$$

$$\int_0^{-1} \frac{u^5}{3} du$$

$$\frac{u^6}{18} \Big|_0^{-1} \quad F(-1) - F(0) = \frac{1}{18} - 0 = \frac{1}{18}$$