Notes 3 **Name ________________________**

Continuity (2.4) and The Intermediate Value Theorem (2.8)

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Continuity

A function *f* is continuous at point *a* if:
I. $f(a)$ exists
II. $\lim_{x\to a} f(x)$ exists
III. $\lim_{x\to a} f(x) = f(a)$

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Intermediate Value Theorem

If *f* is continuous on the closed interval [a,b] and N is a number between *f(a)* and *f(b)*, where $f(a) \neq f(b)$, then there is a number *c* (between a and b) such that $f(c)=N$.

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Determining Continuity: Use the formal **definition of continuity** to determine whether *f* is continuous at *a*. State whether the continuity is jump, removable, or infinite.

1) $f(x) = \dfrac{x^2+8x}{x^2-8x}$

a. $a=0$

b. $a=8$

2) $f(x) = \begin{cases} 2-x & \text{if } x<1 \\ 1 & \text{if } x=1 \\ x^2 & \text{if } x>1 \end{cases}$

a. $a=1$

b. $a=7$

Determining Where a Function is Discontinuous: Determine for what numbers, if any, the given function is discontinuous. State whether the continuity is jump, removable, or infinite.

3) $f(x) = \dfrac{x+2}{x^2-3x-10}$

4) $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x=3 \end{cases}$

Find the constant c that makes $f(x)$ continuous on $(-\infty, \infty)$.

5) $f(x) = \begin{cases} cx + 1 & \text{if } x \le 3 \\ cx^2 - 1 & \text{if } x > 3 \end{cases}$

6) Show that $\frac{t}{t+1}$ takes on the value 0.499 for some t in $[0,1]$.

7) Prove that there is at least one zero for $f(x) = x^2 - 3$ using the Intermediate Value Theorem.