Notes **Name** _____

Separable Differential Equations

Linear Approximations: Find the linear approximation for $f(1.3)$.

1. $\frac{dy}{dx} = 2x, \quad (1,3)$.

$\frac{dy}{dx} = 2$

$f(x) - 3 = 2(x-1)$

$f(x) = 2x + 1$

$f(1.3) = 2(1.3) + 1$

$\boxed{f(1.3) = 3.6}$

2. $\frac{dy}{dx} = \frac{x}{y}, \quad (1,-2)$.

$\frac{dy}{dx} = -\frac{1}{2}$

$f(x) + 2 = -\frac{1}{2}(x-1)$

$f(x) = -\frac{1}{2}x - \frac{3}{2}$

$f(1.3) = -\frac{1}{2}(1.3) - \frac{3}{2}$

$\boxed{f(1.3) = -2.15}$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c$

$y = \pm\sqrt{x^2 + c}$

$-2 = -\sqrt{1 + c}$

$4 = 1 + c$

$y = -\sqrt{x^2 + 3}$

$f(1.3) = -\sqrt{1.69 + 3}$

$\boxed{f(1.3) = -2.166}$

Separable Differential Equations: Find the original function.

3. $\frac{dy}{dx} = x^2 y$

$\frac{dy}{y} = x^2\,dx$

$\int \frac{dy}{y} = \int x^2\,dx$

$\ln|y| = \frac{x^3}{3} + c$

$e^{\ln y} = e^{1/3\,x^3 + c}$

$y = e^{x^3/3} \cdot e^c$

$\boxed{y = ce^{x^3/3}}$

4. $\frac{dy}{dx} = \frac{x}{2y}$

$\int 2y\,dy = \int x\,dx$

$y^2 = \frac{x^2}{2} + c$

$\boxed{y = \sqrt{\frac{1}{2}x^2 + c}}$

Separable Differential Equations: Find the initial value function.

5. $\frac{dy}{dx} = \frac{x^2}{y^2}, \; y(0) = 2$

$\int y^2 dy = \int x^2 dx$

$\frac{y^3}{3} = \frac{x}{3} + c$

$y^3 = 3x^3 + 3c$

$y = \sqrt[3]{3x^3 + c}$

$2 = \sqrt[3]{c}$

$c = 8$

$y = \sqrt[3]{x^3 + 8}$

6. $\frac{dy}{dx} = y(2x+1); \; y(0) = -5$

$\int \frac{dy}{y} = \int (2x+1) dx$

$\ln|y| = x^2 + x + c$

$e^{\ln y} = e^{x^2+x+c}$

$y = Ce^{x^2+x}$

$-5 = Ce^0$

$-5 = C$

$y = -5e^{x^2+x}$