Related Rates and Optimization Review #1

1. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a river. He won't need fence along the river. What are the dimensions of the field that would maximize the area of the field?

2. A cylindrical can is to be made for 1 L ($1000cm^3$) of oil. Find the dimensions that will minimize the cost of material used. (top, bottom and cylinder)

3. Find the point on $y^2=2x$ that is closest to (1,4).

4. Find the area of the largest rectangle that can be inscribed in the parabola $y = 16-x^2$ above the x-axis.

5. An oil spill is growing in a circular shape. The radius is increasing at the rate of 5 miles per day. How fast is the area changing when the circumference is 49π miles?

6. A 17 foot ladder is leaning against a wall. It is sliding down the wall at a rate of 2ft/min.
   (a) How fast is the ladder moving away from the wall when the top is 15 feet from the ground?
   (b) How fast is the area enclosed by the ladder changing when the top is 15 feet from the ground?
   (c) How fast is the angle at the ground changing when the top is 15 feet from the ground?

7. A cone with diameter 8 and height 28 is being filled with water at a rate of 2 cubic feet per second.

   (a) Express the volume of the water as a function of the water level *h*.

   (b) How fast is the level of the oil rising in the cone when h = 4?

   (c) How fast is the radius increasing when h = 4?

   (d) How fast is the exposed surface area (circle) increasing when h = 4?

8. A man observes the launching of a rocket from a distance of 300 feet. The rocket is launched at a speed of 100 ft/sec. How fast is the rocket moving away from the man four seconds after launch? How fast is the angle of vision changing at this same moment?

9. A spherical balloon is being inflated at the rate of $2in^3/sec$. How fast is the radius increasing when the radius is 10 inches?

10. A six foot tall man is walking toward a 10 foot tall light pole at a speed of two feet per second. How fast is the tip of the man's shadow moving when he is eight from the lamp?

1) x=600, y = 1200 2) r=5.419, h=10.839 3) x=2, y=2 4) 49.267 5) dA=245π 6) a)15/4; b) 20.125 or 161/8 ; c)-1/4 7) a) $V = \frac{\pi h^3}{147}$; b) $\frac{49}{8\pi}$; c) $\frac{7}{8\pi}$; d) 1 ft^2/s 8) 80; 3/25 9) $\frac{1}{200\pi}$ 10) -5